

Chapter Eight

Mixed-Frequency Data Models

Tingguo Zheng (zhengtg@gmail.com)

School of Economics & WISE & Chow, Xiamen University

(For Chow & SOE & WISE Msc and PhD Program, 2026-2027, Autumn Semester)

- **Reference:**

- 郑挺国. 宏观计量经济学[M]. 北京: 北京大学出版社, 2026.

- **Outline:**

- Mixed data sampling model
- Mixed-frequency dynamic factor model
- Mixed-frequency VAR model
- Mixed-frequency term structure model

1 Mixed Data Sampling Model

1.1 Bridge equation (Baffigie et al., 2004)

The bridge equation model uses linear regression to bridge high- and low-frequency variables. Let y_t denotes quarterly GDP,

$$y_t = \beta_0 + \lambda y_{t-1} + \beta(L)x_t^Q + \varepsilon_t \quad (1)$$

$$x_t^Q = \omega(L^{1/3})x_t^M = \omega_0 x_t^M + \omega_1 L^{1/3} x_t^M + \cdots + \omega_r L^{r/3} x_t^M \quad (2)$$

where x_t^Q is the temporally aggregated quarterly counterpart of the monthly indicator x_t^M . The weights depend on aggregation properties.

For flow variables: $x_t^Q = \frac{1}{3}x_t^M + \frac{2}{3}x_{t-1/3}^M + x_{t-2/3}^M + \frac{2}{3}x_{t-1}^M + \frac{1}{3}x_{t-4/3}^M$

For stock variables: $x_t^Q = x_t^M + x_{t-1/3}^M + x_{t-2/3}^M$

The forecasting procedure consists of two stages:

1. Quarterly estimation: using historical GDP and aggregated monthly indicators to estimate the bridge regression by OLS.
2. Monthly completion: forecasting unreleased monthly observations using a monthly AR model, aggregate them to the target quarter, and substitute the resulting estimate into the fitted bridge equation.

Example: Q2 GDP Nowcast in May

Suppose monthly industrial production growth is observed for Feb, Mar, Apr. An AR(1) model $\hat{x}_{m+1}^M = \hat{\beta}_0 + \hat{\beta}_1 x_m^M$ predicts May, and June.

For a flow series, the quarterly growth approximation is

$$\hat{x}_{Q2}^Q = \frac{1}{3}\hat{x}_{\text{Jun}}^M + \frac{2}{3}\hat{x}_{\text{May}}^M + x_{\text{Apr}}^M + \frac{2}{3}x_{\text{Mar}}^M + \frac{1}{3}x_{\text{Feb}}^M$$

1.2 Mixed data sampling model (Ghysels et al., 2002)

Mixed data sampling (MIDAS) uses parameterized polynomial weights to incorporate high- and low-frequency data into a single equation.

The basic MIDAS model is specified as

$$y_t = \beta_0 + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_{t-h}^m + \varepsilon_t \quad (3)$$

where $B(L^{1/m}; \boldsymbol{\theta})$ denotes high-frequency variables' lagged structure.

The lag-weight polynomial can be further defined as

$$B(L^{1/m}; \boldsymbol{\theta}) = \sum_{k=1}^K w(k; \boldsymbol{\theta}) L^{(k-1)/m},$$

where K is the number of high-frequency lags and $w(k; \boldsymbol{\theta})$ determines their relative importance.

As in Ghysels et al. (2007), two common weighting schemes are used:

- Exponential Almon polynomial: $w(k; \boldsymbol{\theta}) = \frac{\exp(\theta_1 k + \dots + \theta_Q k^Q)}{\sum_{k=1}^K \exp(\theta_1 k + \dots + \theta_Q k^Q)}$
- Beta polynomial: $w(k; \boldsymbol{\theta}) = \frac{f(\frac{k}{K}; \theta_1, \theta_2)}{\sum_{k=1}^K f(\frac{k}{K}; \theta_1, \theta_2)}$

where $f(x; a, b) = \frac{x^{a-1}(1-x)^{b-1}\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$, $\Gamma(a) = \int_0^\infty e^{-x} x^{a-1} dx$

The parameters are typically estimated using nonlinear least squares.

In practice, parameter restrictions are often imposed to ensure well-behaved lag weights, such as $\theta_1 \leq 300$ and $\theta_2 < 0$ for the exponential Almon polynomial, and $0 \leq \theta_1, \theta_2 \leq 300$ for the Beta polynomial.

However, NLS estimation becomes computationally demanding and potentially unstable as the number of high-frequency predictors and associated weighting parameters increases.

Application 1: Real GDP nowcasting with financial variables

Following Zheng and Shang (2013), the MIDAS model is applied to nowcast China's quarterly real GDP year-on-year growth using four monthly financial indicators.

- Stock market volatility: 1992M9 – 2012M9
- Money supply: 1994M1 – 2012M9
- Term spread: 1996M1 – 2012M9
- Real effective exchange rate (REER): 1995M1 – 2012M9

The quarterly GDP sample spans 1992Q1 – 2012Q3, with both real-time and final-vintage GDP growth data reported (郑挺国和王霞, 2013). Rolling-window estimation is conducted, with an initial window from 1992M7 to 2010M1. Out-of-sample period: 2010Q1 – 2012Q3.

Data visualization

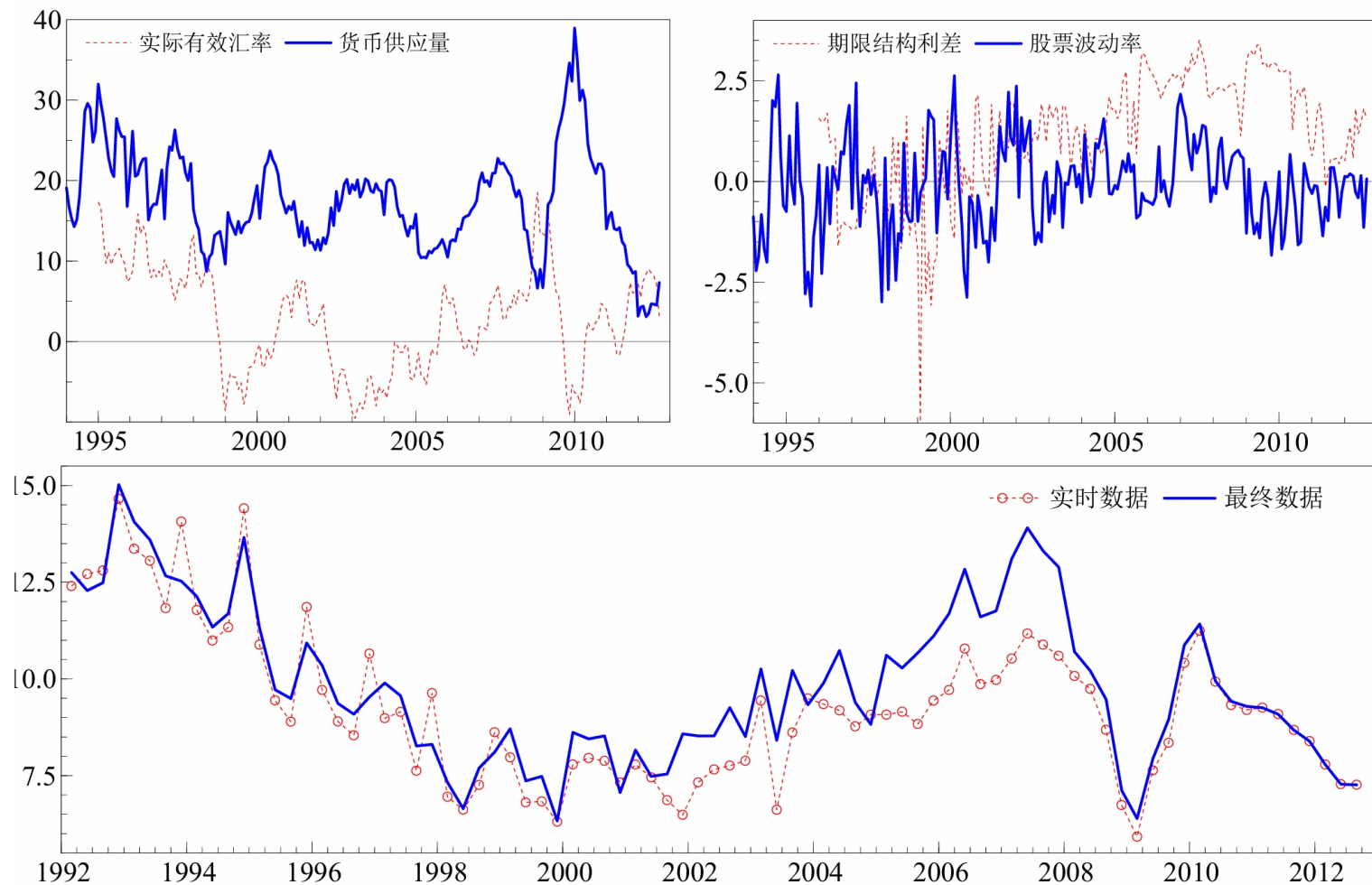


图 8.1 GDP 增长率及金融变量混频数据基本情况

Nowcasting performance of MIDAS models

表 8.1 基于 MIDAS(3, K , 0) 模型对实时数据估计的 MSFE 及 rMSFE

金融变量	$(m, K, 0)$	$MSFE_{beta}$	$MSFE_{almon}$	$rMSFE_{beta}$	$rMSFE_{almon}$
实际有效汇率	(3,3)	1.1525	1.1525	0.9877	0.9877
	(3,6)	1.1607	1.1671	0.9741	0.9794
	(3,12)	1.0965	1.1427	0.9083	0.9466
	(3,18)	1.3967	1.4417	0.9060	0.9352
期限结构利差	(3,3)	1.0718	1.0727	0.9830	0.9839
	(3,6)	0.9812	0.9641	1.0941	1.0750
	(3,12)	0.6869	0.7299	1.0068	1.0699
	(3,18)	0.7310	0.7200	0.9622	0.9477
股票波动率	(3,3)	1.1481	1.1538	0.9883	0.9932
	(3,6)	1.2289	1.2002	1.0040	0.9805
	(3,12)	1.3474	1.3196	1.0079	0.9871
	(3,18)	1.4231	1.2686	0.9983	0.8899
货币供应量	(3,3)	0.7069	0.7204	1.1737	1.1962
	(3,6)	0.6798	0.6695	1.0928	1.0762
	(3,12)	0.6468	0.5969	1.2207	1.1265
	(3,18)	0.5430	0.5999	0.6750	0.7457

注：表中 m 表示季度数据与月度数据的关系其取值为 3， K 表示预测因子的最大滞后阶数，而 $MSFE_{beta}$ 和 $MSFE_{almon}$ 分别表示 Beta 分布和指数 Almon 多项式下得到的 MSFE。

Nowcasting performance of MIDAS-AR models

表 8.2 基于 MIDAS(3, K , 0)-AR(p) 模型的实时数据估计的 MSFE 及 rMSFE

金融变量	(m, K, p)	$MSFE_{beta}$	$MSFE_{almon}$	$rMSFE_{beta}$	$rMSFE_{almon}$
实际有效汇率	(3,3,4)	0.8855	0.8894	0.9959	1.0002
	(3,6,4)	0.8197	0.8602	0.9281	0.9740
	(3,12,4)	0.8026	0.8333	0.8697	0.9030
	(3,18,1)	0.7123	0.7486	0.7141	0.7504
期限结构利差	(3,3,1)	0.6435	0.6433	1.0471	1.0467
	(3,6,1)	0.6078	0.6035	1.0501	1.0426
	(3,12,1)	0.5466	0.5548	0.9794	0.9941
	(3,18,1)	0.5674	0.5356	0.9692	0.9149
股票波动率	(3,3,4)	1.0289	1.0831	0.9584	1.0089
	(3,6,4)	1.0825	1.0767	1.0016	0.9962
	(3,12,1)	1.0807	1.0767	1.0045	1.0162
	(3,18,1)	0.9434	0.9394	0.9978	0.9936
货币供应量	(3,3,4)	0.5640	0.5692	0.9190	0.9275
	(3,6,4)	0.5574	0.5575	0.8476	0.8477
	(3,12,4)	0.7646	0.5664	0.8999	0.6665
	(3,18,4)	0.8308	0.5955	1.0637	0.7625

注：表中 m 表示季度数据与月度数据的关系其取值为 3， K 表示预测因子的最大滞后阶数，而 $MSFE_{beta}$ 和 $MSFE_{almon}$ 分别表示 Beta 分布和指数 Almon 多项式下得到的 MSFE。

表 8.3 MIDAS(m, K, h)-AR(p) 模型下金融指标与投资、消费、出口的预测结果比较

	$K = 3$		$K = 6$		$K = 12$		$K = 18$	
	$h = 0$							
	beta	almon	beta	almon	beta	almon	beta	almon
投 资	1.1071	1.1071	1.0700	1.0532	1.0321	0.9368	0.9545	0.9291
货币供应量	0.5640	0.5692	0.5574	0.5575	0.5664	0.7646	0.8308	0.5955
期限结构利差	0.6435	0.6433	0.6078	0.6035	0.5466	0.5548	0.5674	0.5356
消 费	1.0191	1.0191	1.0726	1.0679	0.9538	0.8442	0.9287	1.0198
股票波动率	1.0289	1.0831	1.0825	1.0767	1.0933	1.0807	0.9434	0.9394
出 口	1.0487	1.0385	1.0451	1.0339	0.9636	0.9906	1.1239	0.9772
实际有效汇率	0.8855	0.8894	0.8197	0.8602	0.8026	0.8333	0.7123	0.7486
	$h = 1$							
	beta	almon	beta	almon	beta	almon	beta	almon
投 资	1.1214	1.1156	1.1278	1.0932	1.0523	1.0568	0.9926	0.9980
货币供应量	0.5574	0.5574	0.6469	0.5650	0.6839	0.6509	1.0110	0.5808
期限结构利差	0.6036	0.5914	0.5681	0.5668	0.5249	0.5406	0.5308	0.5511
消 费	1.3561	1.3561	1.5280	1.4726	1.3338	1.2578	1.2904	1.2828
股票波动率	1.0567	1.0595	1.0479	1.0641	1.0263	0.9300	0.9881	0.8714
出 口	1.0932	1.0932	1.1360	1.0955	1.0068	1.0072	1.1062	0.9975
实际有效汇率	0.8875	0.8991	0.8857	0.8935	0.7871	0.8050	0.7843	0.7993

注：表中 m 表示高频月度频率与低频季度频率的关系始终为 3， K 表示最大滞后阶数， h 表示预测步长，beta 和 almon 分别表示基于加权函数 Beta 和 Almon 多项式的 MIDAS 模型估计的 MSFE。

1.3 Extended MIDAS models

- Asymmetric-MIDAS (Ghysels et al., 2005, 2007)

$$y_{t+1} = \beta_0 + \beta_1(\phi B(L^{1/m}; \boldsymbol{\theta}^-) \mathbf{1}_t^- x_t^{(m)} + (2 - \phi) B(L^{1/m}; \boldsymbol{\theta}^+) \mathbf{1}_t^+ x_t^{(m)}) + \varepsilon_{t+1}$$

- Factor-MIDAS (Marcellino and Schumacher, 2010)

$$y_{t+1} = \beta_0 + \alpha_0 f_t + \alpha_1 B(L^{1/k}; \phi) f_t^{(k)} + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_t^{(m)} + \varepsilon_{t+1}$$

- Autoregressive Distributed Lag-MIDAS (Andreou et al., 2011, 2013)

$$y_{t+1} = \beta_0 + \lambda y_t + \beta_1 B(L^{1/m}; \boldsymbol{\theta}) x_t^{(m)} + \varepsilon_{t+1}$$

- Unrestrict-MIDAS (Foroni et al, 2015)

1.4 Markov-switching MIDAS model

Traditional MIDAS models assume time-invariant parameters.

Guérin and Marcellino (2013) extend the MIDAS framework by allowing model parameters to vary across economic regimes.

$$y_t = \beta_0(S_t) + \beta_1(S_t)B(L^{1/m}; \boldsymbol{\theta})x_{t-h}^{(m)} + \epsilon_t(S_t) \quad (4)$$

where $p_{ij} = \Pr(S_{t+1} = j | S_t = i)$, $\sum_{j=1}^M p_{ij} = 1$, $\forall i, j \in \{1, \dots, M\}$.

The regime-dependent intercept $\beta_0(S_t)$, predictive coefficient $\beta_1(S_t)$, and variance $\sigma^2(S_t)$ capture changes in economic conditions, predictive relationships, and volatility across business cycles.

2 Mixed-Frequency Dynamic Factor Model

2.1 Monthly representation of quarterly variables

Among the N indicators, let N_q be quarterly variables and N_m be monthly variables: $N = N_q + N_m$. Assume that each quarterly variable has a latent monthly series $Y_{q,t}^*$. For a flow variable,

$$Y_{q,t} = 3 \times \frac{1}{3} \sum_{i=1}^3 Y_{q,t-i+1}^*. \quad (5)$$

Following Mariano and Murasawa (2003), the geometric aggregation and its log form are given by

$$Y_{q,t} = 3 \left(\prod_{i=1}^3 Y_{q,t-i+1}^* \right)^{1/3}, \quad \ln Y_{q,t} = \ln 3 + \frac{1}{3} \sum_{i=1}^3 \ln Y_{q,t-i+1}^*. \quad (6)$$

Then, quarterly growth rates can be expressed as weighted averages of the corresponding latent monthly growth rates.

- Quarter-on-quarter growth

$$y_{q,t} = \ln Y_{q,t} - \ln Y_{q,t-3}, \quad y_{q,t}^* = \ln Y_{q,t}^* - \ln Y_{q,t-1}^*. \quad (7)$$

$$y_{q,t} = \frac{1}{3}y_{q,t}^* + \frac{2}{3}y_{q,t-1}^* + y_{q,t-2}^* + \frac{2}{3}y_{q,t-3}^* + \frac{1}{3}y_{q,t-4}^*. \quad (8)$$

- Year-on-year growth

$$y_{q,t} = \ln Y_{q,t} - \ln Y_{q,t-12}, \quad y_{q,t}^* = \ln Y_{q,t}^* - \ln Y_{q,t-12}^*. \quad (9)$$

$$y_{q,t} = \frac{1}{3}y_{q,t}^* + \frac{1}{3}y_{q,t-1}^* + \frac{1}{3}y_{q,t-2}^*. \quad (10)$$

2.2 Mixed-frequency dynamic factor model

Define the vector of latent monthly and observed monthly variables as $\mathbf{y}_t^* = (\mathbf{y}_{q,t}^{*'}, \mathbf{y}_{m,t}')'$. Following King et al. (1991), assume that a common factor drives the joint movements of all indicators:

$$\begin{bmatrix} \mathbf{y}_{q,t}^* \\ \mathbf{y}_{m,t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}_q \\ \boldsymbol{\mu}_m \end{bmatrix} + \boldsymbol{\beta} f_t + \mathbf{u}_t. \quad (11)$$

The common and specific factors follow

$$\phi_f(L) f_t = \nu_{1,t}, \quad \boldsymbol{\Phi}_u(L) \mathbf{u}_t = \boldsymbol{\nu}_{2,t}, \quad (12)$$

$$\begin{bmatrix} \nu_{1,t} \\ \boldsymbol{\nu}_{2,t} \end{bmatrix} \sim N \left(\mathbf{0}, \begin{bmatrix} \sigma_1^2 & \mathbf{0}' \\ \mathbf{0} & \boldsymbol{\Sigma}_{22} \end{bmatrix} \right). \quad (13)$$

Identification restrictions:

$$\beta_1 = 1 \quad \text{or equivalently} \quad \sigma_1 = 1, \quad (14)$$

$$\Phi_u(L) = \text{diag} \{ \Phi_{u,1}(L), \dots, \Phi_{u,N}(L) \}, \quad (15)$$

$$\Sigma_{22} = \text{diag} \{ \sigma_{2,1}^2, \dots, \sigma_{2,N}^2 \}, \quad (16)$$

$$\text{Cov}(u_{i,t}, u_{j,t}) = 0, \quad i \neq j. \quad (17)$$

Using the aggregation rules in equations (8) and (10), the mixed-frequency dynamic factor model can be written for different quarterly growth measures.

- Quarter-on-quarter growth

$$\begin{bmatrix} \mathbf{y}_{q,t} \\ \mathbf{y}_{m,t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}_q \\ \boldsymbol{\mu}_m \end{bmatrix} + \begin{bmatrix} \boldsymbol{\beta}_1 \left(\frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4} \right) \\ \boldsymbol{\beta}_2 f_t \end{bmatrix} + \begin{bmatrix} \frac{1}{3}\mathbf{u}_{1,t} + \frac{2}{3}\mathbf{u}_{1,t-1} + \mathbf{u}_{1,t-2} + \frac{2}{3}\mathbf{u}_{1,t-3} + \frac{1}{3}\mathbf{u}_{1,t-4} \\ \mathbf{u}_{2,t} \end{bmatrix}. \quad (18)$$

- Year-on-year growth

$$\begin{bmatrix} \mathbf{y}_{q,t} \\ \mathbf{y}_{m,t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}_q \\ \boldsymbol{\mu}_m \end{bmatrix} + \begin{bmatrix} \frac{1}{3}\boldsymbol{\beta}_1 (f_t + f_{t-1} + f_{t-2}) \\ \boldsymbol{\beta}_2 f_t \end{bmatrix} + \begin{bmatrix} \frac{1}{3}(\mathbf{u}_{1,t} + \mathbf{u}_{1,t-1} + \mathbf{u}_{1,t-2}) \\ \mathbf{u}_{2,t} \end{bmatrix}. \quad (19)$$

(1) State-space representation

Equations (11) can be written as a linear Gaussian state-space model.

$$y_t = \mu + Hs_t, \quad (20)$$

$$s_t = Fs_{t-1} + G\nu_t, \quad \nu_t \sim N(\mathbf{0}, \Sigma_\nu). \quad (21)$$

The state vector contains the common factor and the specific factors:

$$s_t = (f_t, \dots, f_{t-p^*+1}, u'_t, \dots, u'_{t-q^*+1})'.$$

The transition matrix has a block companion form:

$$F = \begin{bmatrix} F_f & \mathbf{0} \\ \mathbf{0} & F_u \end{bmatrix}, \quad G = \begin{bmatrix} e_1 & \mathbf{0} \\ \mathbf{0} & e_u \end{bmatrix}, \quad \Sigma_\nu = \begin{bmatrix} \sigma_1^2 & \mathbf{0}' \\ \mathbf{0} & \Sigma_{22} \end{bmatrix}.$$

where

$$\mathbf{F}_f = \begin{bmatrix} \phi_{f,1} & \phi_{f,2} & \cdots & \phi_{f,p^*} \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}, \quad \mathbf{F}_u = \begin{bmatrix} \Phi_{u,1} & \Phi_{u,2} & \cdots & \Phi_{u,q^*} \\ \mathbf{I}_N & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_N & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}.$$

The measurement matrix applies the appropriate temporal aggregation weights to the common-factor and specific-factor states. Define the aggregation weights as $\omega_{Q \circ Q} = (\frac{1}{3}, \frac{2}{3}, 1, \frac{2}{3}, \frac{1}{3})'$ and $\omega_{Y \circ Y} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})'$.

The quarterly and monthly measurement equations are

$$\mathbf{y}_{q,t} = \boldsymbol{\mu}_q + \beta_1 \boldsymbol{\omega}' \mathbf{f}_t^* + (\boldsymbol{\omega}' \otimes \mathbf{I}_{N_q}) \mathbf{u}_{1,t}^*, \quad (22)$$

$$\mathbf{y}_{m,t} = \boldsymbol{\mu}_m + \beta_2 \mathbf{f}_t + \mathbf{u}_{2,t}. \quad (23)$$

(2) Missing observations and gaussian likelihood

Let A denote the set of months in which the quarterly variables are missing. The joint probability density function (the likelihood function) is

$$f(\mathbf{y}_T; \boldsymbol{\theta}) = \prod_{t \notin A} f(\mathbf{y}_{q,t}, \mathbf{y}_{m,t}; \boldsymbol{\theta}, \mathbf{y}_{t-1}) \prod_{t \in A} f(\mathbf{y}_{m,t}; \boldsymbol{\theta}, \mathbf{y}_{t-1}). \quad (24)$$

To handle the missing observations, introduce a selection matrix $\mathbf{W}_t$:

$$\mathbf{W}_t \mathbf{y}_t = \mathbf{W}_t \boldsymbol{\mu} + \mathbf{W}_t \mathbf{H} \mathbf{s}_t. \quad (25)$$

Define $\mathbf{y}_t^w = \mathbf{W}_t \mathbf{y}_t$, $\boldsymbol{\mu}_t^w = \mathbf{W}_t \boldsymbol{\mu}$, $\mathbf{H}_t^w = \mathbf{W}_t \mathbf{H}$. Then,

$$\mathbf{y}_t^w = \boldsymbol{\mu}_t^w + \mathbf{H}_t^w \mathbf{s}_t. \quad (26)$$

The Gaussian log-likelihood is

$$\begin{aligned}\ln L(\boldsymbol{\theta}, \mathbf{y}_T^w) = & -\frac{NT}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T \ln |\boldsymbol{\Sigma}_{t|t-1}(\boldsymbol{\theta})| \\ & - \frac{1}{2} \sum_{t=1}^T \boldsymbol{\eta}_{t|t-1}(\boldsymbol{\theta})' \boldsymbol{\Sigma}_{t|t-1}(\boldsymbol{\theta})^{-1} \boldsymbol{\eta}_{t|t-1}(\boldsymbol{\theta}).\end{aligned}\tag{27}$$

(3) Kalman filter and smoothing

Define the conditional state estimate and its covariance, together with the forecast error and its covariance, as

$$\begin{aligned} \mathbf{s}_{t|s} &= E(\mathbf{s}_t \mid \mathbf{y}_s^w; \boldsymbol{\theta}), & \mathbf{P}_{t|s} &= \text{Var}(\mathbf{s}_t \mid \mathbf{y}_s^w; \boldsymbol{\theta}), \\ \boldsymbol{\eta}_{t|t-1} &= \mathbf{y}_t^w - \boldsymbol{\mu}_t^w - \mathbf{H}_t^w \mathbf{s}_{t|t-1}, & \boldsymbol{\Sigma}_{t|t-1} &= \mathbf{H}_t^w \mathbf{P}_{t|t-1} \mathbf{H}_t^{w'}. \end{aligned}$$

Kalman filter recursively performs the following two steps:

(i) Prediction: $\mathbf{s}_{t|t-1} = \mathbf{F} \mathbf{s}_{t-1|t-1},$

$$\mathbf{P}_{t|t-1} = \mathbf{F} \mathbf{P}_{t-1|t-1} \mathbf{F}' + \mathbf{G} \boldsymbol{\Sigma}_\nu \mathbf{G}'.$$

(ii) Updating: $\mathbf{s}_{t|t} = \mathbf{s}_{t|t-1} + \mathbf{K}_t \boldsymbol{\eta}_{t|t-1},$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{K}_t \mathbf{H}_t^w \mathbf{P}_{t|t-1}.$$

Kalman Gain: $\mathbf{K}_t = \mathbf{P}_{t|t-1} \mathbf{H}_t^{w'} (\mathbf{H}_t^w \mathbf{P}_{t|t-1} \mathbf{H}_t^{w'})^{-1}$

After completing the forward filter, the full sample information can be used to obtain the smoothed states. Define the smoothing gain as

$$\mathbf{J}_t = \mathbf{P}_{t|t} \mathbf{F}' \mathbf{P}_{t+1|t}^{-1}.$$

Starting from $t = T - 1$, the smoothed state is obtained recursively:

$$\mathbf{s}_{t|T} = \mathbf{s}_{t|t} + \mathbf{J}_t (\mathbf{s}_{t+1|T} - \mathbf{F} \mathbf{s}_{t|t}).$$

The corresponding smoothed covariance matrix is

$$\mathbf{P}_{t|T} = \mathbf{P}_{t|t} + \mathbf{J}_t (\mathbf{P}_{t+1|T} - \mathbf{P}_{t+1|t}) \mathbf{J}_t'.$$

Therefore,

$$\hat{\mathbf{s}}_{t|T} = E \left(\mathbf{s}_t \mid \mathbf{y}_T^w; \hat{\boldsymbol{\theta}} \right)$$

Application 2: monthly coincident index for china

The application follows Zheng and Wang (2011, 2013).

Quarterly variable: real GDP

Monthly indicators: industrial production, retail sales, fiscal revenue
power generation, electricity consumption, tax revenue
fixed-asset investment, imports and exports, and M1.

- All indicators enter the model as year-on-year growth rates.
- Overall sample: 1979M1 – 2024M12.
- The GDP factor loading is normalized to $\beta_{GDP} = 1$.
- The series are demeaned before estimation.

Parameter estimates

表 8.4 月度混频一致指数

参数	GDP	IP	Sale	INV	CZSR	IMEX	TAX	FDL	ELV	M1
β	1.00	1.523 (0.079)	1.710 (0.145)	2.442 (0.326)	2.486 (0.326)	2.276 (0.287)	2.084 (0.336)	1.373 (0.129)	1.331 (0.190)	0.709 (0.156)
ϕ_f	0.937 (0.016)									
σ_f	1.262 (0.080)									
ϕ_1^u	0.106 (0.095)	0.290 (0.108)	0.908 (0.068)	0.670 (0.050)	0.101 (0.049)	0.408 (0.041)	0.505 (0.056)	0.564 (0.059)	0.501 (0.085)	0.935 (0.050)
ϕ_2^u	0.6552 (0.0998)	-0.233 (0.091)	-0.006 (0.068)	-0.188 (0.050)	0.166 (0.049)	0.404 (0.041)	0.014 (0.055)	0.232 (0.061)	0.178 (0.091)	0.007 (0.051)
σ_u	1.129 (0.113)	1.252 (0.100)	2.205 (0.114)	11.535 (0.420)	16.635 (0.580)	8.145 (0.267)	11.925 (0.438)	2.296 (0.103)	3.891 (0.200)	2.925 (0.102)
$\log L$	-11390.4									

注：小括号内为参数对应的标准差，“ $\log L$ ”表示模型的对数似然值。

Markov mean-switching model

Following Hamilton (1989), the monthly coincident index growth rate f_t is modeled as a two-regime Markov-switching process:

$$f_t = \mu_{S_t} + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_f^2), \quad S_t \in \{1, 2\},$$

where S_t is a latent regime indicator, μ_1 and μ_2 are the mean growth rates in the low-growth and high-growth regimes, respectively.

The transition probabilities and the transition matrix are given by

$$p_{ij} = \Pr(S_t = j \mid S_{t-1} = i), \quad \sum_{j=1}^2 p_{ij} = 1 \quad \text{for each } i$$

Monthly coincident index and growth regimes

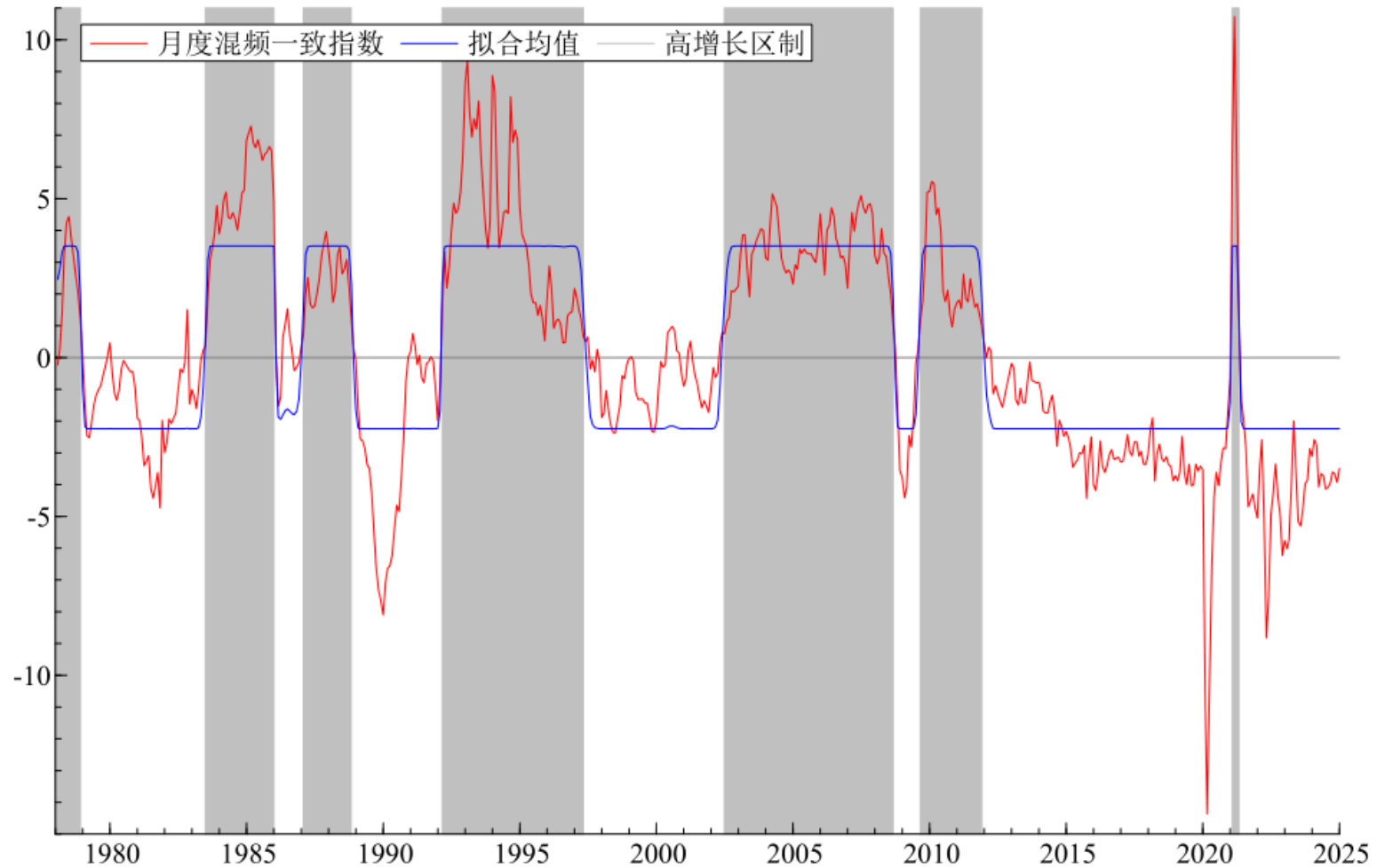


图 8.2 月度混频一致指数

Estimated monthly real GDP growth

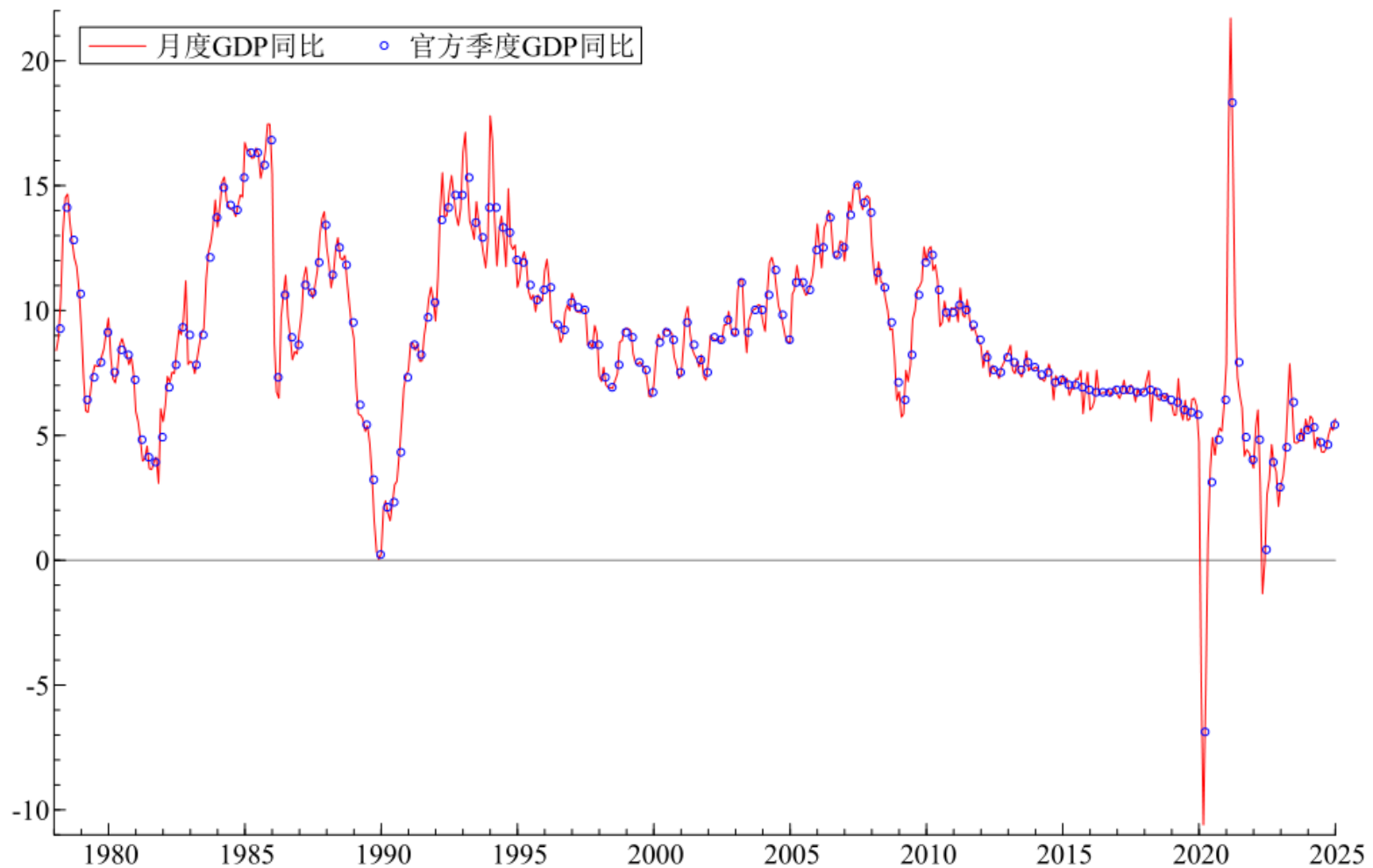


图 8.3 月度 GDP 同比估计值

2.3 Score-Driven mixed-frequency dynamic factor model

Ye and Zheng (2025) extend the mixed-frequency dynamic factor model using the score-driven framework of Creal et al. (2013).

Mixed-frequency aggregation + score-driven factor

The common factor is updated by

$$f_{t+1} = \text{autoregressive component} + \text{scaled conditional score.}$$

- The factor is updated using the conditional score.
- No state-space representation is required.
- MLE remains applicable under non-Gaussian distributions.

Temporal aggregation

Let $y_{i,t} = \ln Y_{i,t} - \ln Y_{i,t-1}$ denote the high-frequency growth rate, and let k_i be the number of high-frequency periods within one low-frequency period. The aggregate low-frequency $\tilde{y}_{i,t}$ is constructed as follows:

$$\text{Stock variables: } \tilde{y}_{i,t} = \begin{cases} \sum_{j=0}^{k_i-1} y_{i,t-j}, & \tilde{y}_{i,t} \text{ is observed,} \\ NA, & \tilde{y}_{i,t} \text{ is unobserved.} \end{cases}$$

Flow variables:

$$\tilde{y}_{i,t} = \begin{cases} \sum_{j=0}^{k_i-1} \frac{j+1}{k_i} y_{i,t-j} + \sum_{j=k_i}^{2k_i-1} \frac{2k_i-j-1}{k_i} y_{i,t-j}, & \tilde{y}_{i,t} \text{ is observed,} \\ NA, & \tilde{y}_{i,t} \text{ is unobserved.} \end{cases}$$

For quarterly variables, $k_i = 3$ and $\omega = (\frac{1}{3}, \frac{2}{3}, 1, \frac{2}{3}, \frac{1}{3})$.

Following King et al. (1991) & Aruoba et al. (2009), the high-frequency growth rate $y_{i,t}$ is driven by a latent common factor, while also capturing idiosyncratic dynamics through its own lagged values:

$$y_{i,t} = \mu_i + \gamma_{i1}y_{i,t-k_i} + \cdots + \gamma_{iq}y_{i,t-k_{iq}} + \beta_i f_t + \varepsilon_{i,t}. \quad (28)$$

Aggregating equation (28) maps this high-frequency structure to the low-frequency domain. The aggregate low-frequency variable satisfies:

$$\tilde{y}_{i,t} = \tilde{\mu}_i + \gamma_{i1}y_{i,t-k_i} + \cdots + \gamma_{iq}y_{i,t-k_{iq}} + \beta_i C_t + E_{i,t}. \quad (29)$$

For stock variables, $C_t = \sum_{j=0}^{k_i-1} f_{t-j}$, $E_{i,t} = \sum_{j=0}^{k_i-1} \varepsilon_{i,t-j}$, $\tilde{\mu}_i = k_i \mu_i$.

For flow variables, $C_t = \sum_{j=0}^{2k_i-1} w_j^i f_{t-j}$, $E_{i,t} = \sum_{j=0}^{2k_i-1} w_j^i \varepsilon_{i,t-j}$.

Mixed-frequency observation equations

Let $\mathbf{y}_t = (\mathbf{y}_t^M, \mathbf{y}_t^{FQ}, \mathbf{y}_t^{SQ})'$, where M , FQ , and SQ denote monthly, quarterly flow, and quarterly stock variables, respectively.

$$\mathbf{y}_t^M = \mathbf{c}^M + \sum_{j=1}^q \Phi_j^M \mathbf{y}_{t-j}^M + \beta^M f_t + \varepsilon_t^M. \quad (30)$$

$$\mathbf{y}_t^{FQ} = \mathbf{c}^{FQ} + \sum_{j=1}^q \Phi_j^{FQ} \mathbf{y}_{t-3j}^{FQ} + \beta^{FQ} C_t^{FQ} + \varepsilon_t^{FQ} + \vartheta \varepsilon_{t-3}^{FQ}. \quad (31)$$

$$\mathbf{y}_t^{SQ} = \mathbf{c}^{SQ} + \sum_{j=1}^q \Phi_j^{SQ} \mathbf{y}_{t-3j}^{SQ} + \beta^{SQ} C_t^{SQ} + \varepsilon_t^{SQ}. \quad (32)$$

where the aggregate factor terms for the quarterly variables are defined as $C_t^{FQ} = \frac{1}{3}(f_t + 2f_{t-1} + 3f_{t-2} + 2f_{t-3} + f_{t-4})$, and $C_t^{SQ} = f_t + f_{t-1} + f_{t-2}$.

Following Creal et al. (2013), the factor dynamics are driven by score,

$$f_{t+1} = \sum_{j=1}^p \rho_j f_{t-j+1} + \alpha s_t, \quad s_t = \mathcal{I}_t^{-\gamma} \nabla_t. \quad (33)$$

The conditional score ∇_t and Fisher information $\mathcal{I}_t$ are given by

$$\nabla_t = \frac{\partial \ln p(\mathbf{y}_t \mid f_t, \boldsymbol{\theta})}{\partial f_t}, \quad \mathcal{I}_t = E_{t-1} (\nabla_t \nabla_t').$$

The parameter γ controls the scaling of the score, yielding the forms $s_t = \nabla_t, \mathcal{I}_t^{-1/2} \nabla_t, \mathcal{I}_t^{-1} \nabla_t$, corresponding to $\gamma = 0, 1/2, 1$, respectively.

The idiosyncratic shocks for each indicator are assumed to follow

$$\varepsilon_{i,t} \sim D_i(0, \sigma_i^2, \phi_i), \quad i = 1, \dots, N.$$

which directly determines the shape of the score function.

Possible choices include

Gaussian, Student- t , skewed Student- t , Gaussian mixture.

Assume the idiosyncratic shocks are **Gaussian**: $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$.

Conditional on f_t , the log-likelihood contribution is

$$\ln p(\mathbf{y}_t \mid f_t, \boldsymbol{\theta}) = -\frac{1}{2} \sum_{i=1}^N \delta_t^i [\ln 2\pi + \ln \sigma_i^2 + \varepsilon_{i,t}^2 \sigma_i^{-2}], \quad (34)$$

where the observation indicator δ_t^i equals 1 if $y_{i,t}$ is observed, and 0 if it is missing. The score and Fisher information can be written as:

$$\begin{aligned} \nabla_t &= \sum_{i=1}^{n_M} \delta_t^i \beta_i \varepsilon_{i,t} \sigma_i^{-2} + \frac{1}{3} \sum_{i=n_M+1}^{n_M+n_{FQ}} \delta_t^i \beta_i \varepsilon_{i,t} \sigma_i^{-2} + \sum_{i=n_M+n_{FQ}+1}^N \delta_t^i \beta_i \varepsilon_{i,t} \sigma_i^{-2}. \\ \mathcal{I}_t &= \sum_{i=1}^{n_M} \delta_t^i \beta_i^2 \sigma_i^{-2} + \frac{1}{9} \sum_{i=n_M+1}^{n_M+n_{FQ}} \delta_t^i \beta_i^2 \sigma_i^{-2} + \sum_{i=n_M+n_{FQ}+1}^N \delta_t^i \beta_i^2 \sigma_i^{-2}. \end{aligned}$$

Assume the idiosyncratic shocks are **Student- t** : $\varepsilon_{i,t} \sim t_{\nu_i}(0, \sigma_i^2)$.

The conditional log-likelihood contribution is

$$\ln p(\mathbf{y}_t \mid f_t, \boldsymbol{\theta}) = \sum_{i=1}^N \delta_t^i \left[\lambda_i - \frac{\nu_i + 1}{2} \ln \left(1 + \frac{\varepsilon_{i,t}^2}{(\nu_i - 2)\sigma_i^2} \right) \right], \quad (35)$$

where $\lambda_i = \ln \Gamma \left(\frac{\nu_i + 1}{2} \right) - \ln \Gamma \left(\frac{\nu_i}{2} \right) - \frac{1}{2} \ln(\nu_i - 2) - \frac{1}{2} \ln \pi - \ln \sigma_i$. The conditional score and Fisher information can be written as:

$$\begin{aligned} \nabla_t &= \sum_{i=1}^{n_M} \delta_t^i \beta_i w_{i,t} \varepsilon_{i,t} \sigma_i^{-2} + \frac{1}{3} \sum_{i=n_M+1}^{n_M+n_{FQ}} \delta_t^i \beta_i w_{i,t} \varepsilon_{i,t} \sigma_i^{-2} + \sum_{i=n_M+n_{FQ}+1}^N \delta_t^i \beta_i w_{i,t} \varepsilon_{i,t} \sigma_i^{-2} \\ \mathcal{I}_t &= \sum_{i=1}^{n_M} \delta_t^i \eta_i \beta_i^2 \sigma_i^{-2} + \frac{1}{9} \sum_{i=n_M+1}^{n_M+n_{FQ}} \delta_t^i \eta_i \beta_i^2 \sigma_i^{-2} + \sum_{i=n_M+n_{FQ}+1}^N \delta_t^i \eta_i \beta_i^2 \sigma_i^{-2}. \end{aligned}$$

where $w_{i,t} = \frac{\nu_i + 1}{(\nu_i - 2) + \varepsilon_{i,t}^2 \sigma_i^{-2}}$, $\eta_i = \frac{(\nu_i + 1)\nu_i}{(\nu_i + 3)(\nu_i - 2)}$.

Estimation and identification of SD-MF-DFM

Let $m = \max(p, 3q, 5)$. The sample log-likelihood is

$$L_T(\boldsymbol{\theta}) = \sum_{t=1}^T \ln p(\mathbf{y}_t \mid \mathcal{F}_{t-1}, \boldsymbol{\theta}) = \sum_{t=1}^T \ln p(\mathbf{y}_t \mid f_t, \boldsymbol{\theta}). \quad (36)$$

Under some regularity conditions (Harvey, 2013; Blasques et al., 2022):

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(\mathbf{0}, -\boldsymbol{\mathcal{H}}^{-1}), \quad \boldsymbol{\mathcal{H}} = -E\left[\frac{\partial^2 L_T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'}\right]. \quad (37)$$

Following Stock and Watson (2016), impose

$$\beta_{\text{GDP}} = 1 \quad \implies \quad f_t \text{ measures monthly macroeconomic conditions.}$$

3 Mixed-Frequency VAR Model

3.1 MIDAS-VAR model (Ghysels, 2016)

MIDAS-VAR jointly models high- and low-frequency variables without aggregating all observations to the lower frequency.

Let $\mathbf{x}_t$ denote the stacked vector of m high-frequency observations and the low-frequency variable within period t ,

$$\mathbf{x}_t = (x_H(t, 1)', \dots, x_H(t, m)', x_L(t)')'.$$

The resulting K -dimensional MIDAS-VAR(p) is

$$\mathbf{x}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{x}_{t-1} + \dots + \mathbf{A}_p \mathbf{x}_{t-p} + \boldsymbol{\varepsilon}_t. \quad (38)$$

where $K = mK_H + K_L$, and K_H , K_L denote the numbers of high- and low-frequency variables, respectively.

Remarks. The reduced-form MIDAS-VAR captures lagged interactions but does not explicitly identify contemporaneous relationships within a low-frequency period. Structural restrictions can incorporate economic timing and information availability.

Three types of restrictions can be considered:

- Within-period transmission: Earlier high-frequency information may influence subsequent observations and low-frequency outcomes.
- Forward-looking effects: Current economic decisions may incorporate expectations of future information or policy responses.
- Exclusion restrictions: Contemporaneous relationships that lack a clear economic rationale are set to zero.

Following the SVAR framework, the MIDAS-VAR can be extended as

$$\mathbf{A}_c \mathbf{x}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{x}_{t-1} + \cdots + \mathbf{A}_p \mathbf{x}_{t-p} + \varepsilon_t, \quad (39)$$

where $\mathbf{A}_c$ captures within-period relationships among high- and low-frequency variables, with the following block structure:

$$\mathbf{A}_c \mathbf{x}_t = \begin{bmatrix} I_{K_H} & \cdots & A_c^{1,m} & A_c^{1,m+1} \\ \vdots & \ddots & \vdots & \vdots \\ A_c^{m,1} & \cdots & I_{K_H} & A_c^{m,m+1} \\ A_c^{m+1,1} & \cdots & A_c^{m+1,m} & I_{K_L} \end{bmatrix} \begin{bmatrix} x_H(t, 1) \\ \vdots \\ x_H(t, m) \\ x_L(t) \end{bmatrix}. \quad (40)$$

Below-diagonal blocks reflect forward information transmission, while above-diagonal blocks may represent anticipatory responses to future economic conditions or policy changes.

An unrestricted stacked VAR can become highly parameterized as the frequency ratio m and lag order p increase. Ghysels (2016) considers a restricted specification with $K_H = K_L = 1$ and demeaned variables. Write the reduced-form system as

$$\mathbf{x}_t = \mathbf{A}_1(\Psi)\mathbf{x}_{t-1} + \cdots + \mathbf{A}_p(\Psi)\mathbf{x}_{t-p} + \varepsilon_t. \quad (41)$$

Its lag-coefficient matrices have the schematic structure

$$\mathbf{A}_1(\Psi) = \begin{pmatrix} H(\rho) & a b(\rho) \\ \omega_1(\gamma)' & \alpha_1 \end{pmatrix}, \quad \mathbf{A}_j(\Psi) = \begin{pmatrix} 0 & 0 \\ \omega_j(\gamma)' & \alpha_j \end{pmatrix}, \quad j \geq 2.$$

- ρ and a govern the ARX(1) dynamics of the high-frequency indicator; α_j are low-frequency autoregressive coefficients.
- MIDAS weights $\omega_j(\gamma)$ are controlled by two shape parameters; the innovation covariance is summarized by three additional parameters.

3.2 Classical MF-VAR model (Mariano and Murasawa, 2010)

Following Mariano and Murasawa (2010), the classical MF-VAR model assumes that all variables follow a latent high-frequency process, while low-frequency variables are observed only periodically.

Let N_1 quarterly variables and N_2 monthly variables constitute an $N = N_1 + N_2$ dimensional system. Quarterly growth is approximated by a weighted average of latent monthly growth rates:

$$y_{t,1} \approx \frac{1}{3}y_{t,1}^* + \frac{2}{3}y_{t-1,1}^* + y_{t-2,1}^* + \frac{2}{3}y_{t-3,1}^* + \frac{1}{3}y_{t-4,1}^*.$$

The mixed-frequency observation equation can be written as

$$\mathbf{y}_t - \boldsymbol{\mu} = H(L)(\mathbf{y}_t^* - \boldsymbol{\mu}^*), \quad H(L) = H_0 + H_1L + H_2L^2 + H_3L^3 + H_4L^4,$$

where $H_j = \text{diag}(\omega_j I_{N_1}, \mathbb{I}(j=0)I_{N_2})$, with $\boldsymbol{\omega} = (\frac{1}{3}, \frac{2}{3}, 1, \frac{2}{3}, \frac{1}{3})$.

Assume the latent monthly variables follow a VAR(p) process:

$$\Phi(L)(\mathbf{y}_t^* - \boldsymbol{\mu}^*) = \mathbf{w}_t, \quad \mathbf{w}_t \sim N(\mathbf{0}, \Sigma). \quad (42)$$

Then, the latent monthly VAR can be expressed in state-space form:

$$\begin{aligned} \mathbf{s}_{t+1} &= A\mathbf{s}_t + B\mathbf{z}_t, \quad \mathbf{z}_t \sim N(\mathbf{0}, I_N), \\ \mathbf{y}_t &= \boldsymbol{\mu} + C\mathbf{s}_t. \end{aligned} \quad (43)$$

where $\mathbf{y}_t$ is the mixed-frequency observation vector, containing missing values, and $\mathbf{s}_t$ is the latent state vector, whose structure depends on the VAR lag order p .

Case 1: $p \leq 5$. The latent state vector contains five consecutive monthly observations, which is $\mathbf{s}_t = (\mathbf{y}_t^* - \boldsymbol{\mu}^*, \dots, \mathbf{y}_{t-4}^* - \boldsymbol{\mu}^*)'$, then

$$A = \begin{bmatrix} \Phi_1 & \cdots & \Phi_p & O_{N \times (5-p)N} \\ I_{4N} & & & O_{4N \times N} \end{bmatrix},$$

$$B = \begin{bmatrix} \Sigma^{1/2} \\ O_{4N \times N} \end{bmatrix}, \quad C = \begin{bmatrix} H_0 & \cdots & H_4 \end{bmatrix}.$$

Case 2: $p > 5$. When the lag order exceeds the temporal aggregation window, the state vector expands to $\mathbf{s}_t = (\mathbf{y}_t^* - \boldsymbol{\mu}^*, \dots, \mathbf{y}_{t-p+1}^* - \boldsymbol{\mu}^*)'$,

$$A = \begin{bmatrix} \Phi_1 & \cdots & \Phi_{p-1} & \Phi_p \\ I_{(p-1)N} & & & O_{(p-1)N \times N} \end{bmatrix},$$

$$B = \begin{bmatrix} \Sigma^{1/2} \\ O_{(p-1)N \times N} \end{bmatrix}, \quad C = \begin{bmatrix} H_0 & \cdots & H_4 & O_{N \times (p-5)N} \end{bmatrix}.$$

3.2 Bayesian MF-VAR model (Schorfheide and Song, 2015)

Like the classical MF-VAR, Schorfheide and Song (2015) model mixed-frequency data through a latent monthly VAR in state-space form, but adopt Bayesian estimation.

Let $\mathbf{x}_t = (\mathbf{x}'_{m,t}, \mathbf{x}'_{q,t})'$ contain observed monthly indicators and latent monthly counterparts of quarterly variables. The VAR(p) process is

$$\mathbf{x}_t = \Phi_1 \mathbf{x}_{t-1} + \cdots + \Phi_p \mathbf{x}_{t-p} + \Phi_c + \mathbf{u}_t, \quad \mathbf{u}_t \sim N(\mathbf{0}, \Sigma). \quad (44)$$

Temporal Aggregation. Instead of directly modeling quarterly growth using five triangular weights, quarterly log-levels are approximated by averaging three latent monthly log-levels:

$$\tilde{\mathbf{y}}_{q,t} = \frac{1}{3} (\mathbf{x}_{q,t} + \mathbf{x}_{q,t-1} + \mathbf{x}_{q,t-2}). \quad (45)$$

Define the latent state vector as $\mathbf{z}_t = (\mathbf{x}'_t, \mathbf{x}'_{t-1}, \dots, \mathbf{x}'_{t-r+1})'$, where $r = \max(p, 3)$. Then, the observation equation is specified as

$$\mathbf{y}_t = M_t \Lambda \mathbf{z}_t, \quad \Lambda = \begin{bmatrix} J_m & O & O & \cdots \\ \frac{1}{3}J_q & \frac{1}{3}J_q & \frac{1}{3}J_q & \cdots \end{bmatrix}, \quad (46)$$

where $J_m = [I_{N_m} \ O]$ and $J_q = [O \ I_{N_q}]$ select the monthly indicators and latent monthly counterparts of quarterly variables, respectively.

The matrix Λ implements temporal aggregation, while the time-varying selection matrix M_t retains only available observations, accommodating quarterly releases and missing monthly data.

Estimation. Following Schorfheide and Song (2015), a Minnesota-style prior is imposed on the VAR coefficients, shrinking the dynamics toward independent random walks. The conjugate prior takes the form

$$p(\Phi, \Sigma \mid \boldsymbol{\lambda}) \sim \text{MNIW}(M, P, S, \nu), \quad (47)$$

where λ controls prior shrinkage, and MNIW denotes the Matrix-Normal Inverse-Wishart distribution.

Following Carter and Kohn (1994), the Gibbs sampler iterates between the following two conditional posterior distributions:

- Step 1: Conditional on the latent monthly states, draw the coefficients and covariance matrix: $p(\Phi, \Sigma \mid Z_{0:T}, Y, \boldsymbol{\lambda})$.
- Step 2: Conditional on the VAR parameters, draw the latent monthly states: $p(Z_{0:T} \mid \Phi, \Sigma, Y)$, using Forward Filtering Backward Sampling.

4 Mixed-Frequency Term Structure Model

4.1 Nelson-Siegel model with macroeconomic variables

Let $y(\tau)$ denote the yield on a government bond with maturity τ . Nelson and Siegel (1987) proposed the following specification:

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right). \quad (48)$$

- β_1 , β_2 , and β_3 jointly determine the shape of the yield curve.
- λ controls the factor loadings at different maturities.

Diebold and Li (2006) allow the Nelson-Siegel factors to evolve over time. For yields observed at N maturities, the measurement equation is

$$\begin{bmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{bmatrix} = \begin{bmatrix} 1 & \frac{1 - e^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1 - e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} \\ 1 & \frac{1 - e^{-\tau_2\lambda}}{\tau_2\lambda} & \frac{1 - e^{-\tau_2\lambda}}{\tau_2\lambda} - e^{-\tau_2\lambda} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1 - e^{-\tau_N\lambda}}{\tau_N\lambda} & \frac{1 - e^{-\tau_N\lambda}}{\tau_N\lambda} - e^{-\tau_N\lambda} \end{bmatrix} \begin{bmatrix} L_t \\ S_t \\ C_t \end{bmatrix} + \begin{bmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{bmatrix}.$$

- L_t , S_t , and C_t denote the level, slope, and curvature factors.
- The factor loadings vary across maturities, whereas the three factors are common to all yields.

The three latent yield-curve factors follow a VAR(1) process:

$$\begin{bmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{bmatrix} + \begin{bmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{bmatrix}. \quad (49)$$

Equivalently, the dynamic Nelson-Siegel model can be written as

$$\begin{aligned} \mathbf{y}_t &= \Lambda \mathbf{f}_t + \boldsymbol{\varepsilon}_t, \\ \mathbf{f}_t - \boldsymbol{\mu} &= A_1(\mathbf{f}_{t-1} - \boldsymbol{\mu}) + \boldsymbol{\eta}_t. \end{aligned} \quad (50)$$

The measurement and state innovations are assumed to satisfy

$$\begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t \end{bmatrix} \sim N \left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} H & 0 \\ 0 & Q \end{bmatrix} \right). \quad (51)$$

To associate yield-curve factors with the macroeconomy, Diebold et al. (2006) incorporate observable monthly macroeconomic variables.

$$\begin{aligned} \mathbf{y}_t^{(m)} &= \Lambda \mathbf{f}_t^{(m)} + \boldsymbol{\varepsilon}_t. \\ \begin{bmatrix} \mathbf{f}_t^{(m)} - \boldsymbol{\mu}_f \\ \mathbf{x}_t^{(m)} - \boldsymbol{\mu}_x \end{bmatrix} &= A_2 \begin{bmatrix} \mathbf{f}_{t-1}^{(m)} - \boldsymbol{\mu}_f \\ \mathbf{x}_{t-1}^{(m)} - \boldsymbol{\mu}_x \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_{f,t}^{(m)} \\ \boldsymbol{\eta}_{x,t}^{(m)} \end{bmatrix}. \end{aligned} \quad (52)$$

The measurement and state innovations are assumed to satisfy

$$\begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t^{(m)} \end{bmatrix} \sim N \left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} H & 0 \\ 0 & \Omega \end{bmatrix} \right). \quad (53)$$

The quarterly and latent monthly variables are linked by

$$\mathbf{z}_{t,S}^{(q)} = \Pi(L) \mathbf{z}_{t,S}^{(m)},$$

where L is the lag operator and $\Pi(\cdot)$ is a polynomial of order P .

The measurement equation is

$$\begin{bmatrix} \mathbf{y}_{t,N}^{(m)} \\ \mathbf{x}_{t,R}^{(m)} \\ \mathbf{z}_{t,S}^{(q)} \end{bmatrix} = \begin{bmatrix} \mathbf{\Lambda}_{N \times 3} & \mathbf{0}_{N \times R} & \mathbf{0}_{N \times SP} \\ \mathbf{0}_{R \times 3} & \mathbf{I}_{R \times R} & \mathbf{0}_{R \times SP} \\ \mathbf{0}_{S \times 3} & \mathbf{0}_{S \times R} & \mathbf{\Pi}_{S \times SP} \end{bmatrix} \begin{bmatrix} \mathbf{f}_t^{(m)} \\ \mathbf{x}_{t,R}^{(m)} \\ \mathbf{z}_{t,S,P}^{(m)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{y,t}^{(m)} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \quad (54)$$

The latent monthly state vector evolves as

$$\begin{bmatrix} \mathbf{f}_t^{(m)} - \boldsymbol{\mu}_f \\ \mathbf{x}_{t,R}^{(m)} - \boldsymbol{\mu}_x \\ \mathbf{z}_{t,S,P}^{(m)} - \boldsymbol{\mu}_z \end{bmatrix} = \boldsymbol{\Phi}^{(m)} \begin{bmatrix} \mathbf{f}_{t-1}^{(m)} - \boldsymbol{\mu}_f \\ \mathbf{x}_{t-1,R}^{(m)} - \boldsymbol{\mu}_x \\ \mathbf{z}_{t-1,S,P}^{(m)} - \boldsymbol{\mu}_z \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta}_{f,t}^{(m)} \\ \boldsymbol{\eta}_{x,t,R}^{(m)} \\ \boldsymbol{\eta}_{z,t,S,P}^{(m)} \end{bmatrix}. \quad (55)$$

The disturbances satisfy

$$\begin{bmatrix} \boldsymbol{\varepsilon}_{y,t}^{(m)} \\ \boldsymbol{\eta}_t^{(m)} \end{bmatrix} \sim N \left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega} \end{bmatrix} \right). \quad (56)$$

In this mixed-frequency measurement equation, $\Pi_{(S \times SP)}$ describes the conversion from latent high-frequency monthly indicators to observed low-frequency quarterly indicators, where $SP = \sum_{s=1}^S p_s$.

The aggregation matrix has the following block-diagonal form:

$$\Pi_{(S \times SP)} = \begin{bmatrix} \Pi_{(1 \times p_1)} & 0_{(1 \times p_2)} & \cdots & 0_{(1 \times p_S)} \\ 0_{(1 \times p_1)} & \Pi_{(1 \times p_2)} & \cdots & 0_{(1 \times p_S)} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{(1 \times p_1)} & 0_{(1 \times p_2)} & \cdots & \Pi_{(1 \times p_S)} \end{bmatrix}.$$

- S is the number of quarterly macroeconomic indicators.
- p_s is the number of latent monthly observations associated with the s -th quarterly indicator.

Consider one monthly and one quarterly macroeconomic variable. The measurement equation becomes

$$\begin{bmatrix} y_t^{(m)} \\ x_t^{(m)} \\ z_t^{(q)} \end{bmatrix} = \begin{bmatrix} \Lambda_{(N \times 3)} & 0_{(N \times 1)} & 0_{(N \times 3)} \\ 0_{(1 \times 3)} & 1 & 0_{(1 \times 3)} \\ 0_{(1 \times 3)} & 0 & \Pi_{(1 \times 3)} \end{bmatrix} \begin{bmatrix} \mathbf{f}_t^{(m)} \\ x_t^{(m)} \\ z_t^{(m)} \\ z_{t-1}^{(m)} \\ z_{t-2}^{(m)} \end{bmatrix} + \begin{bmatrix} \epsilon_{y,t}^{(m)} \\ 0 \\ 0 \end{bmatrix}. \quad (57)$$

- The state vector contains latent monthly counterparts of the quarterly variable to capture its unobserved intra-quarter dynamics.
- For a quarterly growth-rate variable, $\Pi = [1/3 \ 1/3 \ 1/3]$.

And, the state transition equation is

$$\begin{bmatrix} \mathbf{f}_t^{(m)} \\ x_t^{(m)} \\ z_t^{(m)} \\ z_{t-1}^{(m)} \\ z_{t-2}^{(m)} \end{bmatrix} = \begin{bmatrix} \mu_f \\ \mu_x \\ \mu_z \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi_{5 \times 5} & \mathbf{0}_{5 \times 2} \\ \mathbf{0}_{1 \times 4} & 1 & 0 & 0 \\ \mathbf{0}_{1 \times 4} & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{f}_{t-1}^{(m)} \\ x_{t-1}^{(m)} \\ z_{t-1}^{(m)} \\ z_{t-2}^{(m)} \\ z_{t-3}^{(m)} \end{bmatrix} + \begin{bmatrix} \eta_{f,t}^{(m)} \\ \eta_{x,t}^{(m)} \\ \eta_{z,t}^{(m)} \\ 0 \\ 0 \end{bmatrix}. \quad (58)$$

The observation and state innovations satisfy

$$\begin{bmatrix} \varepsilon_{y,t}^{(m)} \\ \eta_t^{(m)} \end{bmatrix} \sim N \left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \Omega \end{bmatrix} \right). \quad (59)$$

The yield-curve state is

$$\mathbf{f}_t^{(m)} = \left(L_t^{(m)}, S_t^{(m)}, C_t^{(m)} \right)'.$$

4.2 Mixed-frequency DSGE Model with yield curve information

The New Keynesian monetary DSGE model describes macroeconomic dynamics through three structural relationships (Woodford, 2003):

- Aggregate supply: New Keynesian Phillips curve
- Aggregate demand: Dynamic Investment-Saving (IS) curve
- Monetary policy: Taylor rule

The conventional three-equation framework focuses on the output gap, inflation, and the short-term interest rate, providing a limited information set for capturing broader macroeconomic dynamics and expectations.

Shang and Zheng (2018) integrate the forward-looking yield information into a mixed-frequency DSGE model to enrich macroeconomic analysis.

The standard New Keynesian Phillips Curve (Woodford, 2003)

$$\begin{aligned}\pi_t &= \beta E_t(\pi_{t+1}) + \kappa(y_t - z_t), \\ z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}\end{aligned}\tag{60}$$

where z_t denotes an AR(1) supply-side shock.

The dynamic Investment-Saving (IS) curve is given by

$$\begin{aligned}y_t &= E_t y_{t+1} - \phi^{-1} [R_t - E_t(\pi_{t+1})] + g_t, \\ g_t &= \rho_g g_{t-1} + \varepsilon_{g,t}\end{aligned}\tag{61}$$

where g_t represents an AR(1) demand-side shock, capturing shifts in preferences or government spending.

Monetary policy follows the Taylor rule:

$$R_t = \rho_R R_{t-1} + (1 - \rho_R) [g_\pi \pi_t + g_y (y_t - z_t)] + \varepsilon_{R,t}. \quad (62)$$

Define the forecast errors of inflation and the output gap as

$$\eta_t^\pi = \pi_t - E_{t-1} \pi_t, \quad \eta_t^y = y_t - E_{t-1} y_t.$$

Equations above form a Linear Rational Expectations (LRE) model:

$$\Gamma_0(\theta) x_t = \Gamma_1(\theta) x_{t-1} + \Psi(\theta) \varepsilon_t + \Pi(\theta) \eta_t. \quad (63)$$

Under the non-explosiveness condition,

$$\begin{aligned} x_t &= \Gamma_1^*(\theta) x_{t-1} + B_1(\theta) \varepsilon_t + B_2(\theta) \eta_t, \\ \eta_t &= \widetilde{M}_\varepsilon \varepsilon_t + M_\varsigma \varsigma_t. \end{aligned} \quad (64)$$

Under equilibrium indeterminacy, set $M_\varsigma = 1$, $\widetilde{M} = M^*(\theta) + M$.

The optimal value of $M^*(\theta)$ is

$$M^*(\theta) = [B_2(\theta)' B_2(\theta)]^{-1} B_2(\theta)' [B_1(g(\theta)) - B_1(\theta)] .$$

The state-vector solution becomes

$$x_t = \Gamma_1^*(\theta) x_{t-1} + B_M(\theta) \nu_t, \quad M = (M_z, M_g, M_R)' .$$

M captures uncertainty in the transmission of structural shocks.

DSGE model with yield curve information

Following Diebold et al. (2006), the yield curve is characterized by three latent factors: $f_t = (L_t, S_t, C_t)'$, where L_t , S_t , and C_t represent the level, slope, and curvature of the yield curve, respectively.

The level factor captures long-term yields, while the slope factor reflects the short-long yield spread. Under the Nelson-Siegel factor representation, the short-term interest rate satisfies $R_t = L_t + S_t$.

To link the yield curve with macroeconomic conditions,

$$L_t = \mu_L + \varphi_{LL}L_{t-1} + \varphi_{LR}R_{t-1} + \varphi_{LC}C_{t-1} + \varphi_{Ly}Y_{t-1} + \varphi_{L\pi}\pi_{t-1} + \varepsilon_{L,t},$$

$$C_t = \mu_C + \varphi_{CL}L_{t-1} + \varphi_{CR}R_{t-1} + \varphi_{CC}C_{t-1} + \varphi_{Cy}Y_{t-1} + \varphi_{C\pi}\pi_{t-1} + \varepsilon_{C,t}.$$

This specification incorporates yield-curve dynamics into DSGE model through the short-term interest rate and macroeconomic variables.

Define the augmented state vector as $\mathbf{x}_t^A = (L_t, C_t, \mathbf{x}_t')'$. The DSGE model with yield-curve information is

$$\mathbf{x}_t^A = \mathbf{\Gamma}_0^A(\boldsymbol{\theta}) + \mathbf{\Gamma}_1^A(\boldsymbol{\theta})\mathbf{x}_{t-1}^A + \mathbf{B}^A(\boldsymbol{\theta})\boldsymbol{\nu}_t^A, \quad (65)$$

where $\mathbf{B}^A(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_M(\boldsymbol{\theta}) \end{bmatrix}$, $\boldsymbol{\nu}_t^A = (\varepsilon_{L,t}, \varepsilon_{C,t}, \boldsymbol{\nu}_t)'$.

Government bond yields at different maturities satisfy

$$\mathbf{b}_t = \mathbf{b}^A + \mathbf{\Lambda} \mathbf{f}_t + \boldsymbol{\varepsilon}_t, \quad (66)$$

where $\mathbf{b}_t = (b_t(\tau_1), \dots, b_t(\tau_N))'$, $b_i^A = \pi^A + R^A$. Following Diebold and Li (2006), $\mathbf{\Lambda}$ contains the Nelson-Siegel factor loadings.

Assume quarterly GDP is the geometric average of the corresponding monthly GDP values:

$$Y_t^{(q)} = (Y_t^{(m)} Y_{t-1}^{(m)} Y_{t-2}^{(m)})^{1/3}. \quad (67)$$

Following Mariano and Murasawa (2003b) and Zheng and Wang (2013),

$$y_t^{(q)} = \frac{1}{3}y_t + \frac{1}{3}y_{t-1} + \frac{1}{3}y_{t-2}. \quad (68)$$

Mixed-frequency DSGE state-space model

Using $R_t = L_t + S_t$, the measurement equation is

$$\begin{bmatrix} \mathbf{b}_t \\ \tilde{\pi}_t \\ y_t^{(q)} \end{bmatrix} = \begin{bmatrix} \mathbf{b}^A \\ \pi^A \\ 0 \end{bmatrix} + \mathbf{H}^A \mathbf{x}_t^{A,M} + \boldsymbol{\varepsilon}_t^M, \quad (69)$$

where $\mathbf{x}_t^{A,M} = (L_t, C_t, \mathbf{x}_t', y_{t-1}, y_{t-2})'$, $\boldsymbol{\varepsilon}_t^M = (\boldsymbol{\varepsilon}_t', 0, 0)'$.

The state equation is extended to include the lagged monthly output-gap components:

$$\mathbf{x}_t^{A,M} = \begin{bmatrix} \boldsymbol{\Gamma}_0^A(\boldsymbol{\theta}) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Gamma}_1^A(\boldsymbol{\theta}) & \mathbf{0} \\ \mathbf{e}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_2 \end{bmatrix} \mathbf{x}_{t-1}^{A,M} + \begin{bmatrix} \mathbf{B}^A(\boldsymbol{\theta}) \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\nu}_t^A. \quad (70)$$

Bayesian estimation

Because the mixed-frequency DSGE model contains many parameters, estimation is conducted using Bayesian MCMC methods. The likelihood function allowing for both determinate and indeterminate solutions is

$$\ell(\theta, M, \sigma_{\varsigma} \mid Y^T) = \ell^D(\theta \mid Y^T)I\{\theta \in \Theta_D\} + \ell^I(\theta, M, \sigma_{\varsigma} \mid Y^T)I\{\theta \in \Theta_I\}.$$

where M and σ_{ς} are parameters associated with sunspot shocks under indeterminacy. $I\{\cdot\}$ is an indicator function.

The likelihood is obtained from the mixed-frequency state-space model.

The Bayesian estimation proceeds as follows:

1. Solve the linear rational expectations model using the method of Lubik and Schorfheide (2003).
2. Construct the maximum likelihood function from the mixed-frequency state-space model.
3. Use optimization algorithm to maximize the log-likelihood function and obtain the initial estimate $\hat{\theta}$.
4. Compute the Hessian matrix at $\hat{\theta}$ and use singular value decomposition to obtain its inverse, denoted by $\hat{\Sigma}$.
5. Given an initial calibration parameter c_0 , draw the initial value:

$$\theta^{(0)} \sim N(\hat{\theta}, c_0^2 \hat{\Sigma}).$$

For $s = 1, \dots, N$, given the current draw $\theta^{(s-1)}$,

- (1) Generate a candidate parameter vector: $\nu \sim N(\theta^{(s-1)}, c^2 \hat{\Sigma})$
- (2) Accept the candidate ν with probability: $\min\{1, r(\theta^{(s-1)}, \nu \mid y^T)\}$
where $r(\theta^{(s-1)}, \nu \mid y^T) = \frac{\ell(y^T \mid \nu) p(\nu)}{\ell(y^T \mid \theta^{(s-1)}) p(\theta^{(s-1)})}$.
- (3) If the draw is accepted, then $\theta^{(s)} = \nu$.
- (4) Otherwise, retain the previous draw: $\theta^{(s)} = \theta^{(s-1)}$.

The parameter c controls the dispersion of the proposal distribution.

Posterior probability of indeterminacy

For a rational expectations model that permits both determinate and indeterminate solutions, the posterior probability of indeterminacy is defined as

$$\pi^I = \frac{p_I(Y^T)}{p_I(Y^T) + p_D(Y^T)}. \quad (71)$$

The marginal data density under each type of equilibrium is

$$p_s(Y^T) = \int \ell(\theta | Y^T) p(\theta) \cdot I\{\theta \in \Theta_s\} d\theta, \quad s \in \{D, I\}.$$

- $p_D(Y^T)$ and $p_I(Y^T)$ denote the marginal data densities under determinate and indeterminate solutions.
- Following Lubik and Schorfheide (2003), π^I is used to assess the equilibrium state of the economy.

Data and variable construction

The empirical analysis combines three types of macro-financial data:

- Government bond yields (monthly, 2002M1 – 2013M12)

Chinese interbank government bond yields at 11 maturities: 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, and 120 months.

- Inflation (monthly, 2002M1 – 2013M12)

Constructed from seasonally adjusted CPI using TRAMO/SEATS, with annualized monthly inflation defined as $\tilde{\pi}_t = 12 \ln(P_t/P_{t-1})$.

- Output gap (quarterly, 1992Q1 – 2013Q4)

Derived from quarterly real GDP using the Hodrick-Prescott filter.

$$y_t^{(q)} = \ln(Y_t^{(q)}) - \ln(Y_t^{(q,HP)}), \quad Y_t^{(q,HP)} : \text{potential quarterly output.}$$

Prior distributions of parameters

表 8.5 参数先验分布设定形式

第一组参数					第二组参数				
参数	值域	分布	超参1	超参2	参数	值域	分布	超参1	超参2
β	$[0, 1]$	Beta	0.5	0.2	σ_c	$[0, +\infty)$	InvGamma	0.4	4.0
κ	$[0, +\infty)$	Gamma	0.2	0.1	φ_{Lt}	$[0, 1]$	Beta	0.5	0.2
ϕ	$[0, +\infty)$	Gamma	2.0	0.5	φ_{2c}	$(-\infty, +\infty)$	Normal	0.0	1.0
π	$[0, +\infty)$	Gamma	2.0	0.5	φ_L	$(-\infty, +\infty)$	Normal	0.0	1.0
R	$(-\infty, +\infty)$	Normal	0.0	1.0	φ_k	$(-\infty, +\infty)$	Normal	0.0	1.0
ρ_A	$[0, 1]$	Beta	0.5	0.2	φ_z	$(-\infty, +\infty)$	Normal	0.0	1.0
g_x	$[0, +\infty)$	Gamma	1.5	0.25	φ_{2L}	$(-\infty, +\infty)$	Normal	0.0	1.0
g_r	$[0, +\infty)$	Gamma	0.5	0.25	φ_{cc}	$(-\infty, +\infty)$	Normal	0.0	1.0
ρ_r	$[0, 1]$	Beta	0.6	0.1	φ_q	$[0, 1]$	Beta	0.5	0.2
ρ_g	$[0, 1]$	Beta	0.7	0.1	φ_Q	$(-\infty, +\infty)$	Normal	0.0	1.0
$\rho_{z,x}$	$[-1, 1]$	Normal	0.0	1.0	φ_{2cc}	$(-\infty, +\infty)$	Normal	0.0	1.0
M_t	$(-\infty, +\infty)$	Normal	0.0	1.0	σ_z	$[0, +\infty)$	InvGamma	0.4	4.0
M_z	$(-\infty, +\infty)$	Normal	0.0	1.0	σ_c	$[0, +\infty)$	InvGamma	0.4	4.0
M_r	$(-\infty, +\infty)$	Normal	0.0	1.0	φ_c	$[0, 1]$	Beta	0.5	0.2
σ_r	$[0, +\infty)$	InvGamma	0.4	4.0	σ_a	$[0, +\infty)$	InvGamma	0.4	4.0
σ_i	$[0, +\infty)$	InvGamma	0.4	4.0	μ_L	$(-\infty, +\infty)$	Normal	0.0	1.0
σ_g	$[0, +\infty)$	InvGamma	0.4	4.0	μ_c	$(-\infty, +\infty)$	Normal	0.0	1.0

注：逆伽马分布（InvGamma）的概率密度函数可表示为 $IG(\sigma; s, r) = \frac{r^s}{\Gamma(s)} \sigma^{-(s+1)} e^{-r/\sigma^2} / (2\sigma^2)$ （其中 s 为形状参数， r 为尺度参数， $\Gamma(\cdot)$ 为伽马函数）。

Parameter estimation

表 8.6 基于利率期限结构信息的混频DSGE模型参数估计

参数	均值	标准差	95%估计区间	参数	均值	标准差	95%估计区间
β	0.7810	0.1224	[0.5410, 0.9805]	σ_c	0.1032	0.0393	[0.0428, 0.1744]
κ	0.8711	0.0175	[0.8426, 0.8989]	φ_{2c}	0.8778	0.0387	[0.8035, 0.9507]
ϕ	1.7219	0.2718	[1.2742, 2.1991]	φ_{Lc}	-0.0239	0.0670	[-0.1597, 0.1020]
π^A	4.3296	1.0164	[2.2620, 6.1116]	φ_L	0.0145	0.0223	[-0.0286, 0.0573]
R^A	-0.3876	0.6864	[-1.6932, 0.9852]	φ_Q	0.4830	0.4922	[-0.4779, 1.4623]
ρ_k	0.9503	0.0089	[0.9314, 0.9649]	φ_{Lx}	-0.1848	1.0348	[-2.0174, 1.8473]
g_x	0.6380	0.1372	[0.3731, 0.8867]	φ_{Qz}	-0.0127	0.0411	[-0.0941, 0.0629]
g_r	0.4103	0.2011	[0.0539, 0.7773]	φ_{cc}	0.8813	0.0456	[0.8008, 0.9627]
ρ_c	0.9165	0.0325	[0.8486, 0.9705]	φ_Q	-0.0239	0.0217	[-0.0694, 0.0181]
ρ_g	0.7811	0.0301	[0.6605, 0.8975]	φ_{Qc}	-0.2431	0.5514	[-1.3109, 0.7884]
ρ_d	0.1842	0.2726	[-0.3869, 0.7475]	φ_z	1.6424	1.0276	[-0.3865, 3.7039]
M_d	0.2961	0.9520	[-1.5055, 2.2632]	σ_z	0.0291	0.0029	[0.0233, 0.0350]
M_z	-1.0356	0.4168	[-1.9230, -0.2811]	σ_c	0.0409	0.0051	[0.0321, 0.0513]
M_r	-0.4069	0.9338	[-2.1175, 1.1573]	λ	0.0162	0.0027	[0.0113, 0.0212]
σ_r	0.0284	0.0024	[0.0233, 0.0335]	μ_L	0.0146	0.0003	[0.0141, 0.0152]
σ_d	0.3097	0.0410	[0.2048, 0.3941]	μ_c	0.0077	0.0071	[-0.0047, 0.0224]
σ_g	0.1090	0.0376	[0.0534, 0.1941]	μ_c	0.0033	0.0081	[-0.0099, 0.0230]
π^A				1.0000			
Log-MDD				3843.39			
接受概率				0.4544			

Impulse-response analysis

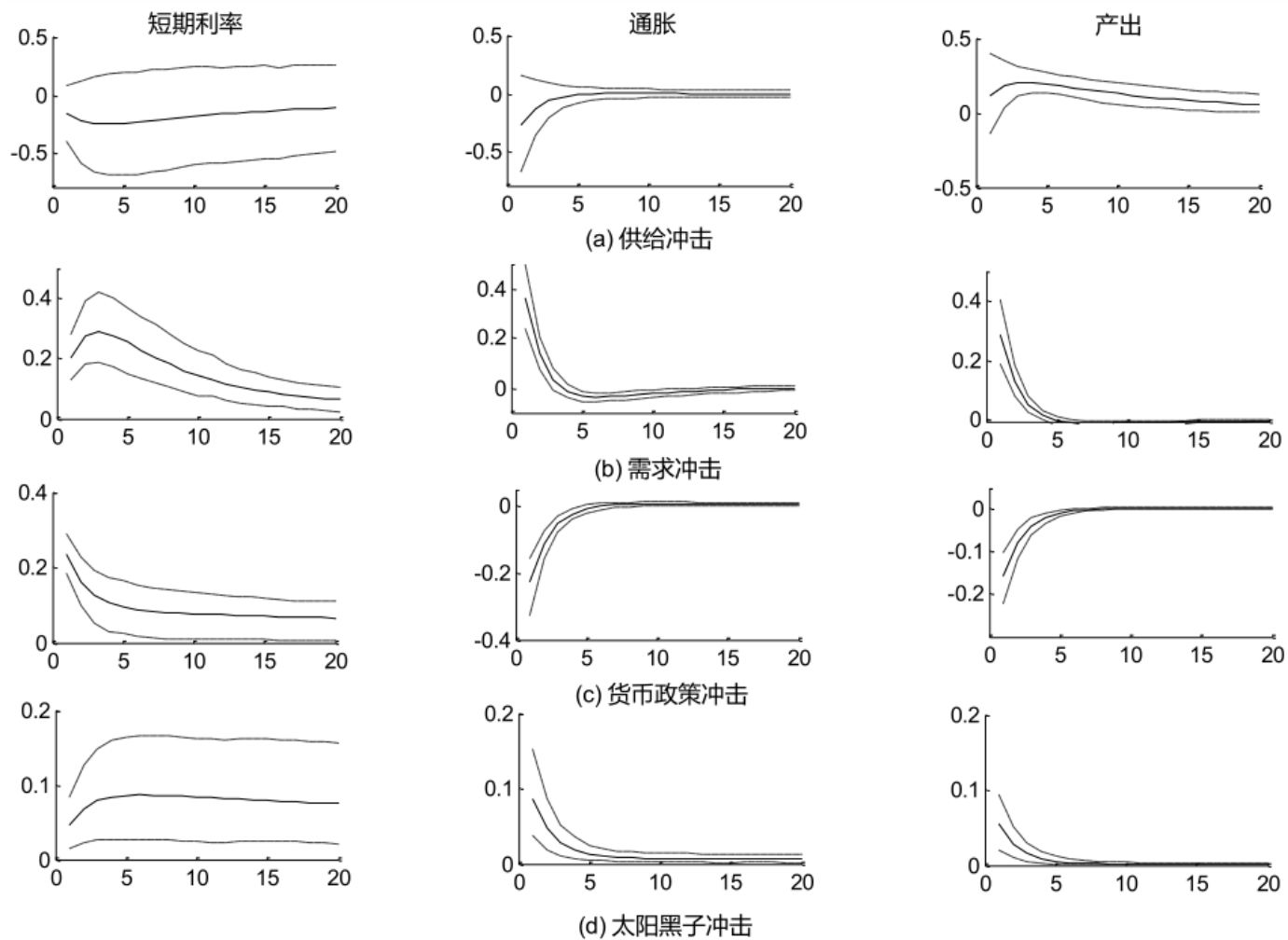


图 8.4 脉冲响应估计结果