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Mixed-frequency SV model for stock volatility and macroeconomics[☆]

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ABSTRACT

This paper develops a stochastic volatility-mixed frequency data sampling (SV-MIDAS) model with low frequency macro variables and further extends it to an asymmetric SV-MIDAS model. Empirical study is then implemented on both Chinese and U.S. stock markets. Our results show that the SV-MIDAS model is useful to identify the macroeconomic volatility source of stock volatility and improve the in-sample fitting performance. Moreover, the out-of-sample forecast performances of SV-MIDAS model are significantly superior to that of traditional SV model for both Chinese and U.S. stock markets. In particular, among the macroeconomic variables, the Composite Leading Indicator has the best forecast performance. In addition, we find that the asymmetric SV-MIDAS model is applicable for capturing leverage effects in both stock markets and it outperforms the corresponding benchmark model in the in-sample fitting.

1. Introduction

The volatility forecast is important for asset pricing and risk management. One of the most popular volatility models is the generalized autoregressive conditional heteroskedasticity (GARCH) family of models (Engle, 1982; Bollerslev, 1986; Engle et al., 2013). As an alternative approach, the stochastic volatility (SV) model proposed by Taylor (1994) has been developed in financial econometrics (Alizadeh et al., 2002; Jensen and Maheu, 2010; Kanaya and Kristensen, 2016; Kastner et al., 2017). In fact, comparing with the GARCH models, the SV model can naturally matched with some financial theory such as continuous time asset pricing model. Recently, many literatures apply the SV model and its extensions to estimate and forecast volatility (Ding and Vo, 2012; Chan and Grant, 2016; Takahashi et al., 2016; Peiris et al., 2017).

Although SV models offer a framework for volatility prediction, only one unobservable factor drives the volatility process in traditional SV models. Obviously, this single factor specification fails to link other important factors with the volatility (Shang and Liu, 2017). It has become a barrier for the SV models to improve the fitting and forecast performance. Instead, many researches, such as Engle and Lee (1999), have shown that the volatility process is driven by different factors. Furthermore, Engle and Rangel (2008) point out that the volatility research should pay more attention to the macroeconomic source of the stock volatility. Chen et al. (2017) find that the conditional volatility of stock pricing factors is significantly related to economic uncertainty. Meanwhile, the related financial theory also shows that a close link exists between the macro economy and stock market volatility (Bansal and Yaron, 2004; Wachter, 2013). These studies suggest that macroeconomic variables could be seen as one of the components or forces that drive the volatility in SV model.

It is well known that the macroeconomic variables are observed at the lower frequency than the stock market variables. To introduce macroeconomic variables into the volatility model, several mixed frequency volatility models have been proposed in recent years (Engle and Rangel, 2008; Engle et al., 2013). Engle and Rangel (2008) first propose a Spline-GARCH model that connects high-frequency volatility with low-frequency realized volatility. It is noteworthy that Engle et al. (2013) further propose the GARCH-MIDAS (mixed-frequency data sampling) model. The GARCH-MIDAS model allows us to extract long-term and short-term component. And the low frequency macroeconomic variables usually explain this long-term component.

The GARCH-MIDAS model decomposes the stock volatility into two

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different components, which helps to identify the macroeconomic source of stock volatility. Moreover, this approach leads naturally to a forecasting tool for volatility (Becker and Clements, 2007). Asgharian et al. (2013) have showed that the GARCH-MIDAS model with low-frequency macroeconomic information might improve its prediction ability, especially for the long-term variance component. Zheng and Shang (2014) propose a multiple factor GARCH-MIDAS model and find that it shows better performance for forecasting the volatility of Chinese stocks. As far as we know, under the framework of GARCH family model, there have been a lot of researches on the mixed-frequency volatility model with macro variables. However, there are few studies on how to build a mixed-frequency SV model with low-frequency macro variables.

In this paper, we develop a mixed-frequency SV model with macro variables that is referred to as the SV-MIDAS model. Shang and Liu (2017) have built the SV model with mixed frequency information. Beyond Shang and Liu (2017), this paper pays more attention to the rule of some crucial macroeconomic variables. And we further extend the asymmetric SV-MIDAS or ASV-MIDAS model. Specifically, we decompose the volatility into a stochastic component and a deterministic (stable) component. The stochastic component maintains the same form as traditional SV models, whereas the stable component is driven by a low-frequency macro variable. This mixed-frequency specification helps to gain insight into macroeconomic sources of volatility and then improve the volatility forecast.

Other contributions can be summarized from two aspects. First, the in-sample fitting and out-of-sample forecast performance of the proposed model are investigated for both Chinese and United States stock markets. We choose both the low-frequency volatility measure (e.g., monthly realized range) and a macroeconomic variable (e.g., Composite Leading Indicator,[1] CPI and M1) as the low-frequency variable. The results show that the SV-MIDAS model outperforms the traditional SV model in the in-sample fitting and out-of-sample performances. Moreover, the volatility of macroeconomic fundamentals has a positive effect on the stable component of stock market volatility. This implies that the SV-MIDAS model can identify the macroeconomic volatility source of stock volatility.

Second, we extend the SV-MIDAS model to include a leverage effect, referred to as the ASV-MIDAS model and its empirical application are also discussed. Our empirical results show there is a weaker leverage effect exists in China's stock market, which is consistent with the results from Shen and Zheng (2009) and Ouyang et al. (2014). In the presence of leverage effect, the volatility of the macroeconomic fundamentals still shows a positive effect on the stable component. Furthermore, we find that the ASV-MIDAS model outperforms the traditional SV model in in-sample fitting. But the out-of-sample forecast results for the ASV-MIDAS model are worse than those of the SV-MIDAS model.

The remainder of this paper is organized as follows. In Section 2, we introduce the model specifications and estimation approach. In Section 3, this paper presents the empirical mixed frequency data information. The empirical results are shown in Section 4. Section 5 concludes the paper.

2. The methodology

2.1. Traditional SV model

Following Harvey et al. (1994), Sandmann and Koopman (1998), and so on, the traditional SV model is given by:

$$r_i = e^{\frac{1}{2}h_i}\varepsilon_i \tag{1}$$

$$h_{i+1} = \mu + \varphi(h_i - \mu) + \eta_i \tag{2}$$

$$\varepsilon_i \sim N(0,1) \tag{3}$$

$$\eta_i \sim N\left(0, \sigma_\eta^2\right) \tag{4}$$

where r_i is the log return at the i-th day; $h_i \equiv \ln(\sigma_i^2)$ is the log variance; $\mu$is intercept of Eq (2) and $\varphi$is coefficient of Eq (2). $\varepsilon_i$is a Gaussian white noise process with unit variance. $\eta_i$is a stochastic process with variance σ_η^2.

For the quasi-maximum likelihood estimation suggested by Harvey et al. (1994), the basic SV model can be rewritten in the following state space form:

$$y_i = h_i + \xi_i \tag{5}$$

$$h_{i+1} = \mu + \varphi(h_i - \mu) + \eta_i \tag{6}$$

where$y_i = \ln(r_i^2)$,$\xi_i = \ln(\varepsilon_i^2)$; ξ_i is assumed to follow the log-chi-square distribution with one degree of freedom, i.e., $\ln(\chi_1^2)$, with its mean being -1.2704 and variance being $\frac{1}{2}\pi^2$; $\eta_i \sim N(0, \sigma_\eta^2)$; and ξ_i and η_i are mutually independent.

2.2. SV-MIDAS model

For notational simplicity, we mark the variable with an appropriate frequency symbol. Denote $r_{i,t}$ as the log return rate on day t during month (quarterly, year) i. Let N_t as the number of days exist in period t. Referring to Engle et al. (2013), the equation for the log return is specified as follows:

$$r_{i,t} = \sigma_{i,t}\sqrt{\tau_t}\varepsilon_{i,t} \tag{7}$$

where $\sigma_{i,t}^2\tau_t$ is the variance and consists of two components: $\sigma_{i,t}^2$ and τ_t.The error term $\varepsilon_{i,t}$ is assumed to follow the standard normal distribution, i.e., $\varepsilon_{i,t}|\psi_{i-1,t} \sim N(0,1)$.

The first component $\sigma_{i,t}$ is viewed as a stochastic component, which is driven by day-to-day liquidity concerns and possibly other short-term shocks. Let $h_{i,t} = \ln \sigma_{i,t}^2$, which can be expressed as follows:

$$h_{i+1,t} = \tilde{\varphi} h_{i,t} + \eta_{i,t} \tag{8}$$

where $\eta_i \sim N(0, \sigma_\eta^2)$,$\varepsilon_{i,t}$ and $\eta_{i,t}$ are mutually independent.

The second component τ_t is considered a stable component determined by the low-frequency macroeconomic or financial variable. The MIDAS method, proposed by Ghysels et al. (2004), is used to construct the stable component equation. Following Engle et al. (2013), the stable component τ_t driven by the low-frequency macroeconomic variable can be expressed as follows:

$$\ln \tau_t = m + \theta \sum_{p=1}^{P} \phi_p(\omega_1, \omega_2) X_{t-p}^n, \tag{9}$$

where m is a constant, θ is the response to the MIDAS structure, P is a maximum lag order defined as the MIDAS lag year (Engle et al., 2013), and X_t^n is a normalized low frequency macroeconomic variable. We denote ω_1 and $\omega_2$are parameters of weighting function. The weighting or smoothing function $\phi_p(\omega_1,\omega_2)$is often defined as the "Beta" lag structure:

$$\phi_p(\omega_1, \omega_2) = \frac{f(p/P, \omega_1, \omega_2)}{\sum_{p=1}^{P} f(p/P, \omega_1, \omega_2)}, \tag{10}$$

where

$$f(x, a, b) = \frac{x^{a-1}(1-x)^{b-1}\Gamma(a+b)}{\Gamma(a) + \Gamma(b)}. \tag{11}$$

[1] Composite Leading Indicator is the index to reflect the future state of macroeconomic. It is often published by official statistical department.

Equations (7)–(11) form the SV-MIDAS model with low frequency variable. This mixed frequency model links the low frequency macro variables with high frequency log return. The macro variables have primarily effect on stable component of the stock volatility.

In the literature, the low-frequency variable is often selected as a long-run volatility measure (Engle et al., 2013; Zheng and Shang, 2014). For example, this variable is sometimes measured using realized volatility$RV_t = \sum_{i=1}^{N_t} r_{i,t}^2$, and thus X_{t-k}^n is the normalized log realized volatility. An alternative measure is the realized range volatility based on the price range information,[2] which can be defined as

$$RG_t = \frac{1}{4*\ln(2)} \sum_{i=1}^{N_t} (\ln(H_{i,t}) - \ln(L_{i,t}))^2 \tag{12}$$

where $H_{i,t}$ and $L_{i,t}$ represent the highest and lowest price on day i during month (quarterly, year) t, respectively.

More importantly, we include a macroeconomic variable into the SV-MIDAS model, which can be completed by replacing X_t^n with this corresponding macroeconomic variable. This type of model helps us investigate the macroeconomic source of stock volatility. It also helps us to find the contribution of the macroeconomic variable to volatility forecasting.

In order to estimate the parameters easily, we could rewrite the above SV-MIDAS model into approximate model,[3] see Kim et al. (1998) and Nakajima and Omori (2009). Let$y_{i,t}^* = y_{i,t} - \ln(\tau_t)$, this approximate model is expressed as follows:

$$y_{i,t}^* = h_{i,t} + \xi_{i,t}^*, \; h_{i+1,t} = \tilde{\varphi} h_{i,t} + \eta_{i,t} \tag{13}$$

where. $\xi_{i,t}^* = \ln(\varepsilon_{i,t}^2)$

Conditional on the indicators$s_i \in \{1, 2, \dots K\}$, Equation (13) can be further rewritten by:

$$\begin{pmatrix} y_{i,t}^* \\ h_{i,t+1} \end{pmatrix} = \begin{pmatrix} h_{i,t} \\ \tilde{\varphi} h_{i,t} \end{pmatrix} + \begin{pmatrix} \xi_{i,t}^* \\ \eta_{i,t} \end{pmatrix} \tag{14}$$

$$\left\{ \begin{pmatrix} \xi_{i,t}^* \\ \eta_{i,t} \end{pmatrix} \middle| s_i = k \right\} = \begin{pmatrix} m_k + \nu_k z_{i,t}^{(1)} \\ \sigma_\eta z_{i,t}^{(2)} \end{pmatrix} \tag{15}$$

where both $z_{i,t}^{(1)}$ and $z_{i,t}^{(2)}$ follow the standard normal distribution with zero mean and unity variance.

2.3. ASV-MIDAS model

We can further extend the SV-MIDAS model to take into account the leverage effect. This extension leads to the asymmetric SV-MIDAS or ASV-MIDAS model. Many studies have found an asymmetric feature—a leverage effect—in stock market volatility (e.g., Nelson, 1991; Harvey and Shephard, 1996; Yu, 2005; Omori et al., 2007). Yu (2005) pointed out that the asymmetric SV model is often formulated in terms of stochastic differential equations.

In continuous-time, we have a logarithmic asset price $p(i,t)$ and the corresponding volatility$\sigma^2(i,t)$. The asymmetric SV model can then be specified as follows:

$$dp(i,t) = \sigma^2(i,t) dB_1(i,t) \tag{16}$$

$$d \ln \sigma^2(i,t) = \alpha_0 + \alpha_1 \ln \sigma^2(i,t) + \sigma_\eta dB_2(i,t) \tag{17}$$

where, $B_1(i,t)$and$B_2(i,t)$are two Brownian motions. $corr(dB_1(i,t), dB_2(i,t)) = \rho$. If the correlation coefficient ρ is negative, then a leverage effect exists (Yu, 2005; Omori et al., 2007).

These equations are often discretized using the Euler-Maruyama approximation method. Then we have the following discrete-time ASV model.

$$r_{i,t} = \sigma_{i,t} \varepsilon_{i,t} \tag{18}$$

$$\ln \sigma_{i+1,t}^2 = \alpha_0 + \tilde{\varphi} \ln \sigma_{i,t}^2 + \eta_{i,t} \tag{19}$$

$$\begin{pmatrix} \varepsilon_{i,t} \\ \eta_{i,t} \end{pmatrix} \sim N(0, \Sigma), \Sigma = \begin{pmatrix} 1 & \rho\sigma_\eta \\ \rho\sigma_\eta & \sigma_\eta^2 \end{pmatrix}, \tag{20}$$

where $r_{i,t} = p(i+1,t) - p(i,t)$, $\varepsilon_{i,t} = B_1(i+1,t) - B_1(i,t)$, $\eta_{i,t} = \sigma_\eta(B_2(i+1,t) - B_2(i,t))$, and $corr(\varepsilon_{i,t}, \eta_{i,t}) = \rho$, and $\tilde{\varphi} = 1 + \alpha_1$.

The specification of ASV-MIDAS model is also same as SV-MIDAS model. The log return in ASV-MIDAS model is also specified as follows:$r_{i,t} = \sigma_{i,t}\sqrt{\tau_t}\varepsilon_{i,t}$. But the ASV-MIDAS model is different from the SV-MIDAS model by its specification of the correlation between $\varepsilon_{i,t}$ and $\eta_{i,t}$.

Similar to SV-MIDAS model, let$y_{i,t}^* = y_{i,t} - \ln(\tau_t)$. Following Nakajima and Omori (2009), we also rewrite the ASV-MIDAS model as the following approximate model:

$$\begin{pmatrix} y_{i,t}^* \\ h_{i,t+1} \end{pmatrix} = \begin{pmatrix} h_{i,t} \\ \tilde{\varphi} h_{i,t} \end{pmatrix} + \begin{pmatrix} \xi_{i,t}^* \\ \eta_{i,t} \end{pmatrix} \tag{21}$$

$$\left\{ \begin{pmatrix} \xi_{i,t}^* \\ \eta_{i,t} \end{pmatrix} \middle| s_i = k, d_i \right\} = \begin{pmatrix} m_k + \nu_k z_{i,t}^1 \\ d_i \rho \sigma_\eta \left(a_k + b_k \nu_k z_{i,t}^1\right) \exp(m_k/2) + \sigma_\eta \sqrt{1-\rho^2} z_{i,t}^2 \end{pmatrix} \tag{22}$$

where bothaville the standard normal distribution with zero mean and unity variance, and $d_i = sign(y_{i,t})$.

2.4. MCMC estimation approach

One of the most popular estimation methods for SV model is the Markov Chain Monte Carlo (MCMC) approach. Following Kim et al. (1998) and Nakajima and Omori (2009), we approximate the log chi-square distribution using a K-component mixture of the normal distribution, and then estimate the mixture approximation model with the MCMC method.

To estimate SV-MIDAS model, we assume the distribution of $\xi_{i,t}^*$ in Equation (14) is a mixture of the normal distribution. Then we have:

$$f\left(\xi_{i,t}^*\right) = \sum_{k=1}^{K} q_k f_N \left(\xi_{i,t}^* \middle| m_k, \nu_k^2\right) \tag{23}$$

The distribution of $\xi_{i,t}^*$ is a mixture of K-component Gaussian densities with component probability q_k. According to Nakajima and Omori (2009), we set$K = 10$. Table 1 provides the weights, means, and variances of normal distribution mixtures.

Based on this approximated model, we present the Bayesian MCMC method for estimating the SV-MIDAS model. More details on the posterior distribution of parameters and the related sampling process are shown the Appendix.

To facilitate parameter estimation of ASV-MIDAS model, the distribution of $\xi_{i,t}^*$ in Equation (21) is also the mixture of the normal distribution. Referring to Nakajima and Omori (2009), we also approximate the $\ln(\chi_1^2)$ with 10-component normal distribution. The related

[2] Parkinson (1980) advocated using range information to estimate volatility. Modeling volatility based on range information has been a topic of increasing focus by researchers.

[3] The 'approximation' of the model means that the log chi-square distribution is approximated by a mixture of K-component Gaussian densities with component probability.

Table 1
Mixtures of normal distributions for the SV-MIDAS model.

k	q_k	m_k	ν_k^2
1	0.0061	1.9268	0.1127
2	0.0478	1.3474	0.1779
3	0.1306	0.7350	0.2677
4	0.2067	0.0227	0.4061
5	0.2272	−0.8517	0.6270
6	0.1884	−1.9728	0.9858
7	0.1205	−3.4679	1.5747
8	0.0559	−5.5525	2.5450
9	0.0158	−8.6838	4.1659
10	0.0012	−14.6500	7.3334

Note: Source from Nakajima and Omori (2009). q_k is the weight of the k-th normal distribution.$m_k$is the mean of the k-th normal distribution.ν_k^2denote the variance of the k-th normal distribution.

Table 2
The mixture of normal distributions for the ASV-MIDAS model.

k	q_k	m_k	ν_k^2	a_k	b_k
1	0.0061	1.9268	0.1127	1.0142	0.5071
2	0.0478	1.3474	0.1779	1.0225	0.5112
3	0.1306	0.7350	0.2677	1.0340	0.5170
4	0.2067	0.0227	0.4061	1.0521	0.5260
5	0.2272	−0.8517	0.6270	1.0815	0.5408
6	0.1884	−1.9728	0.9858	1.1311	0.5656
7	0.1205	−3.4679	1.5747	1.2175	0.6088
8	0.0559	−5.5525	2.5450	1.3745	0.6873
9	0.0158	−8.6838	4.1659	1.6833	0.8416
10	0.0012	−14.6500	7.3334	2.5010	1.2505

Note: Source from Nakajima and Omori (2009).

information for ASV-MIDAS model is shown in Table 2.

3. The data

The data of the Chinese and United States stock markets are selected for empirical study. We collect the daily stock indices and low frequency macroeconomic variables, such as Composite Leading Indicator, CPI, and M1.

First, the daily prices of stock indices are used to compute the daily returns, monthly realized volatility and monthly realized range volatility. We choose the close, high, and low prices of both the Shanghai Composite Index (SH) and the S&P 500 (SP500) as the raw data. The data are sampled from January 3, 1994 to June 30, 2015. The sample sizes of the Shanghai Composite Index and the S&P 500 are 5244 and 5,216, respectively. We calculate the log returns using the close price ($P_{i,t}$), i.e., $r_{i,t} = \ln P_{i,t} - \ln P_{i-1,t}$, and then compute the monthly realized volatility $RV_t = \sum_{i=1}^{N_t} r_{i,t}^2$. We also calculate the monthly realized range volatility $RG_t = (4\ln 2)^{-1}\sum_{i=1}^{N_t}[\ln(H_{i,t}) - \ln(L_{i,t})]^2$ as the proxy variable of low-frequency volatility. The raw data are available from the WIND database.

Second, following Shyu and Hsia (2008), we choose Composite Leading Indicator as the proxy for macroeconomic fundamentals. Zheng and Wang (2013) pointed out that the Composite Leading Indicator can capture China's business cycle. Some studies considered industrial production (IP) and other indexes as proxies of GDP or macroeconomic fundamentals (Engle et al., 2013). However, IP cannot fully reflect the macroeconomic state and business cycle. We also take the CPI as a proxy for inflation that shows the influence of the price index on volatility, and we take M1[4] as a proxy for monetary policy that may reflect the mechanism of monetary policy to financial volatility. The sample period is from January 1994 to June 2015. Raw data are from the China National Bureau of Statistics (CNBS), the U.S. Bureau of Labor Statistics and the Organization for Economic Cooperation and Development (OECD).

Fig. 1 plots the time series data including the log return, RV, RG, and macroeconomic variables.

4. Empirical analysis

4.1. Priors

To estimate the SV-MIDAS model by MCMC approach, we should set the prior information of parameters. The prior distributions of parametersφ, $\tilde{\varphi}$and σ_η are shown below. The prior distribution of φ and $\tilde{\varphi}$ are the beta distribution and that of $\sigma_\eta$is the inverse gamma distribution. More specifically, for the Chinese stock market, the prior distributions of φ and $\tilde{\varphi}$ are $Beta(50, 1.5)$, and that of σ_η is $IG(0.5\nu_0, 0.5\nu_1)$, where $\nu_0 = 50$, and$\nu_1 = 0.01\nu_0$. For the U.S. stock market, the prior distributions of φ and $\tilde{\varphi}$ are $Beta(20, 1.5)$, and σ_η follows $IG(0.5\nu_0, 0.5\nu_1)$, where $\nu_0 = 5$, and $\nu_1 = 0.01\nu_0$. The total number of MCMC sampling of the posterior distribution is 10,000. And we discarded the initial 2000 samples.

4.2. Estimation results

We estimate the SV-MIDAS(P) models with different measures and different lag orders of P. Empirical studies, such as Engle et al. (2013), usually use MIDAS lag years to reflect the lag period. For example, $P = 1$ suggests one MIDAS lag year or, put differently, the SV-MIDAS (1) model will contain the low frequency observations of last 12 months. Similarly, in the SV-MIDAS (2) model, the low frequency information of last 24 months will be included. Moreover, we construct the SV-MIDAS(P)-RG model using the monthly range volatility and the SV-MIDAS(P)-RV model with the monthly realized volatility.

Table 3 reports the parameter estimation of the SV-MIDAS and traditional SV models for the Chinese stock market. To compare the in-sample fitting results, we also calculate the relative root-mean-square error (*rRMSE*), which is defined as follows[5]:$rRMSE = RMSE_a/RMSE_0$, where $RMSE_0$ is the root-mean-square error based on the traditional SV model, and $RMSE_a$ is the root-mean-square error based on SV-MIDAS model. If the *rRMSE* is less than 1, the *RMSEa* will be less than $RMSE_0$. It means that the SV-MIDAS model can improve the in-sample fitting performance of the benchmark model.

From Table 3, we observe the following results. First, the parameter estimates of θ are significantly positive. It implies that both monthly range volatility and realized volatility shows significant positive influence on the stable component. This result is similar to that of Engle et al. (2013); Zheng and Shang (2014); Shang and Liu (2017). Second, the parameter estimates of optimal weight ω_1 reveal the gradual decay trend in the weight function.[6] The parameter estimate of φ is significantly positive, suggesting that the volatility process has strong autocorrelation. The parameter estimate of $\tilde{\varphi}$ is also significantly positive. But the autocorrelation of is $\tilde{\varphi}$less than that of φ. Third, the *rRMSE* results of both SV-MIDAS-RV and SV-MIDAS-RG model are less than 1. This implies that the in-sample fitting performance of the mixed-frequency SV model outweighs that of the traditional SV model. One reason is that the new time-varying stable component is identified via low frequency variable in SV-MIDAS model. Therefore, we suggest that the improvements of the in-sample fitting benefits from the low frequency information in SV-MIDAS model. Fourth, the SV-MIDAS-RG model performs better than the SV-MIDAS-RV model, which shows that the range volatility contributes more to the mix-frequency SV model than the realized volatility.

[4] Because of the absence of samples for M2 in China, this paper considers M1 as a proxy for a monetary supply.

[5] $T^{-1}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)(T^{-1}\sum_{t=1}^{T}(\hat{\sigma}_t^2-\sigma_t^2)(T^{-1}\sum_{t=1}^{T}(\hat{\sigma}_t^2-\sigma_t^2)^2)^{1/2}$, where $\hat{\sigma}_t^2$is the estimated conditional variance with SV model, $\sigma_t^2$is the proxy of volatility calculated from daily range information.

[6] As suggested in Engle et al. (2013), we estimate the parameter ω_1 in Eq. (10) and set $\omega_2 = 1$.

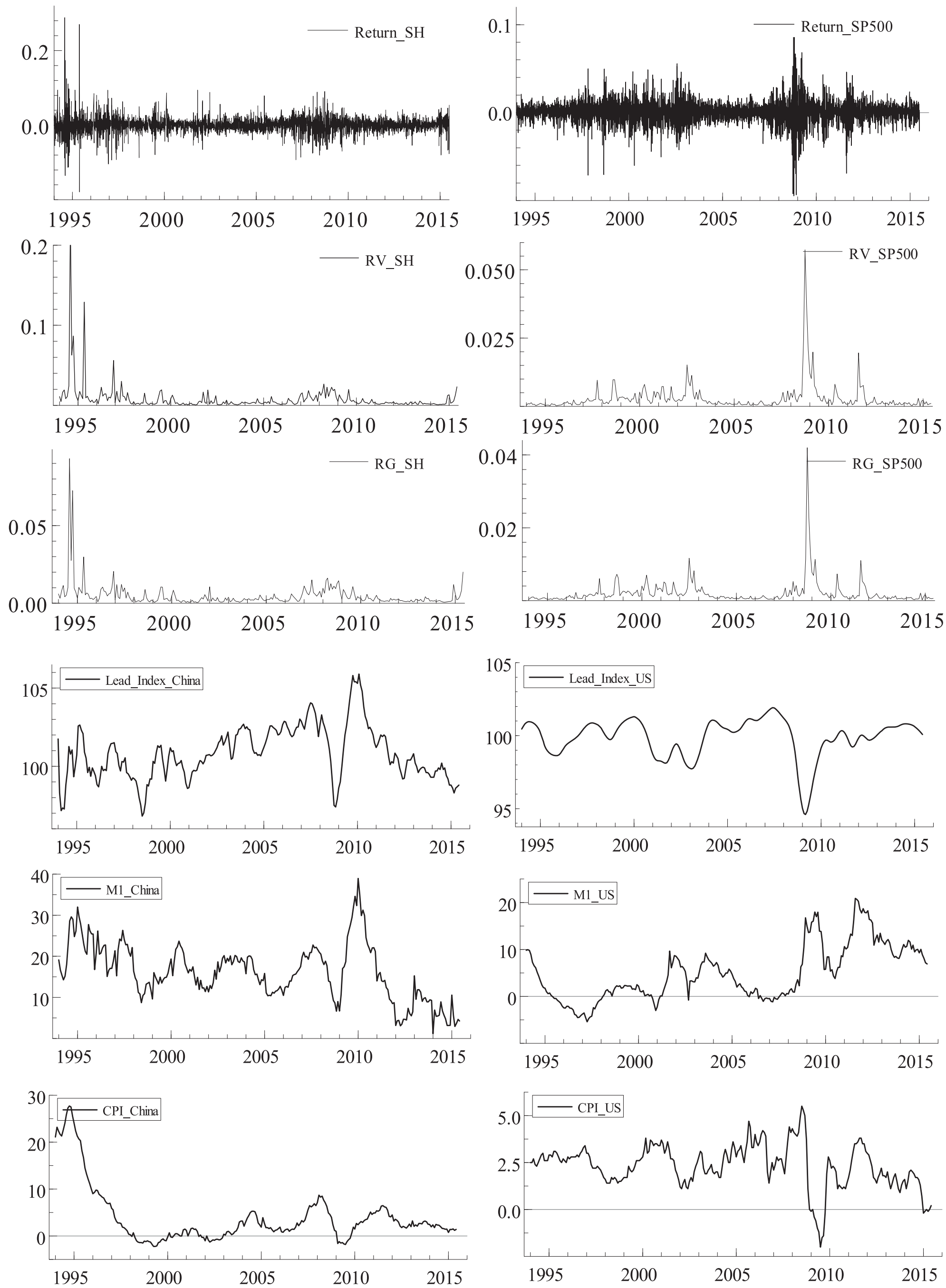

Fig. 1. Log Return and Macro Variables of both China and the United States.

Next we extract the time-vary behavior of stable component. The time-varying stable component is an important feature for the SV-MIDAS model. We can measure the time-varying feature of stable component via Equation (9). This paper takes $\sqrt{\tau_t}$ as the stable component of the daily log return. Meanwhile, the conditional variance is represented by $\sqrt{\sigma_{t,i}^2 \times \tau_t}$. Their estimated results about the Chinese stock market are plotted in Fig. 2.

From Fig. 2, we obtain the following observations. First, the stable component is smoother than the conditional variance for both SV-MIDAS-RV and SV-MIDAS-RG model. However, the stable component shows a co-movement tendency with the conditional variance. Second, the time-varying feature of stable component is identified in both SV-MIDAS-RV and SV-MIDAS-RG model. But the traditional SV model fails to measure this behavior. We suggest that this time-varying stable component should play a crucial rule for volatility fitting and forecast. The results in Table 3 have shown that the estimation of stable component helps to improve the in-sample fitting performance. Moreover, the discovery of the time-varying features also helps us reveal the new volatility mechanism and then improve volatility forecast.

To test the reliability of these results, we apply the SV-MIDAS model to the S&P 500 index. Table 4 reports the estimated results. First, most estimated parameters are statistically significant. For example, the estimates of θ are also significantly positive, which is similar to that in Table 3. That is, a positive effect of low frequency volatility on the stable component is also shown in the U.S. stock market. Second, using the results of the ***rRMSE***, we find that the SV-MIDAS model also shows the better in-sample fitting performance in the U.S. stock market. Third, different from China, the performance of SV-MIDAS-RG (2) does not

Table 3
Estimated results of the SV-MIDAS model (Shanghai Composite Index).

	BSV	SV-MIDAS (1)-RG	SV-MIDAS (2)-RG	SV-MIDAS (1)-RV	SV-MIDAS (2)-RV
φ	0.9623	–	–	–	–
	[0.9488, 0.9742]				
$\tilde{\varphi}$	–	0.9434	0.9484	0.9455	0.9385
		[0.9214, 0.9615]	[0.9282, 0.964]	[0.9213, 0.9647]	[0.9223, 0.9641]
μ	−8.4955	–	–	–	–
	[-8.7066, −8.2887]				
σ_η	0.2747	0.2561	0.2390	0.2305	0.2421
	[0.2346, 0.3160]	[0.2130, 0.3027]	[0.1993, 0.2840]	[0.1927, 0.2738]	[0.1986, 0.2878]
m	–	−8.9801	−8.9504	−9.1348	−8.9584
		[-9.0478, −8.9251]	[-9.0205, −8.8610]	[-9.1802, −9.0915]	[-9.0402, −8.8936]
θ	–	0.6614	0.9273	0.7708	0.9043
		[0.6066, 0.7324]	[0.8806, 0.9720]	[0.7011, 0.8246]	[0.8626, 0.9611]
ω_1	–	3.3845	7.0972	2.5673	6.3376
		[3.3286, 3.4488]	[7.0576, 7.1360]	[2.5021, 2.6739]	[6.2873, 6.4015]
rRMSE	–	0.8596	0.8415	0.8827	0.8655

Note:BSV is the traditional SV model. The reported parameter estimates are means of the posterior distribution. The value in parentheses is the 95% confidence interval. *rRMSE* is the relative *RMSE* for a comparison of the in-sample fit between a SV-MIDAS model and the traditional SV model.

outweigh that of SV-MIDAS-RV (2) for the United States.

The above investigation shows that the mixed-frequency model can identify the time-varying stable component. That component is primarily affected by low-frequency volatility. However, these results provide limited information for explaining the macroeconomic sources of volatility. Following Engle et al. (2013) and Zheng and Shang (2014), we introduce different macroeconomic variables[7] to describe the stable component of daily stock volatility. Because considerable studies, such as Hamilton and Lin (1996) and Wachter (2013), have shown a close relationship between the macroeconomic economy and stock market volatility, we introduce three crucial macroeconomic variables Composite Leading Indicator, CPI and M1) into the SV-MIDAS model.[8]

Table 5 reports the results of the mixed-frequency SV model with three different macroeconomic variables. To save space, we only report the results of the SV-MIDAS (2) model because this model always performs better than the SV-MIDAS (1) model. Moreover, for ease of comparison, we report the estimated results for both the Chinese and the U.S. stock markets.

The results are presented as follows. First, the parameter estimate of θ in the SV-MIDAS (2)-Lead model is significantly positive. It shows that the volatility of the macroeconomic fundamentals has a positive effect on the stable component of stock market volatility. This finding is similar to that of Engle et al. (2013) and Zheng and Shang (2014). In addition, the increase in the volatility of both the CPI and M1 also has a positive effect on the stable component. Second, the values of *rRMSE* is always less than 1. These suggest that the in-sample fitting performances of the SV-MIDAS model outperform those of the traditional SV model. Moreover, we find the greater improvement in the fitting performance of the SV-MIDAS model with macroeconomic variables in China, especially when using the SV-MIDAS (2)-Lead model. In contrast, such an improvement is not obvious for the U.S. stock market. This finding shows that the SV-MIDAS model with macroeconomic variables is more effective in China, an emerging market. Among the macroeconomic variables, the Composite Leading Indicator exhibits the best performance for volatility modeling.

Why do macroeconomic variables play different rule between the Chinese and U.S. stock market? We believe that one of the reasons is rationality and efficiency of the stock market. For the U.S. stock market, it has fully developed and consists of institutional investors who are more rational. So it is more efficient than other stock markets. Instead, the stock market in China is an emerging market. It consists of many individual investors who are less rational than institutional investors. When the macroeconomic shocks occur in these two stock markets, the situations reflecting these two stock markets are different. For the U.S. stock market, the stock price has contained more macroeconomic shocks information since the rationality and efficiency of the stock market. At this time, macroeconomic shocks show weaker effect to the stock price. However, the Chinese stock market is more sensitive to macroeconomic shocks because the market is less efficient and rational. Therefore, macroeconomic shocks have played a more regulatory role in China's stock market volatility.

4.3. In-sample fitting of ASV-MIDAS model

In this section, we report the estimated results of the ASV-MIDAS model using the MCMC method.[9] As shown Table 6, we observe the following results. First, the estimates of ρ are significantly negative, implying that a leverage effect exists in both stock markets. In contrast, a weaker leverage effect exists in China's stock market compared with that in the U.S. stock market. One reason is that China's stock market has a price limit, the maximum decline in a stock price is limited to 10% of its last close price. This regulation weakens the leverage effect in China's stock market. In fact, the leverage effect in China's stock market varies form period to period. Shen and Zheng (2009) use the data form from 1990 to 2009 and find the anti-leverage effect of both Shanghai and Shenzhen Index. Ouyang et al.(2014) and Jiang et al.(2017) point out that Before the year 2000, the Shanghai and Shenzhen markets exhibited the anti-leverage effect. After 2000, however, it gradually changed to the leverage effect. Considering the data sample, our results are consistent with that of Shen and Zheng (2009) and Ouyang et al.(2014). In sum, there is a weak leverage effect exists in China's stock market.

Second, compared with the SV-MIDAS model, the parameter estimates of m, θ and ω_1 are almost the same. In the case of leverage effect, the volatility of the macroeconomic fundamentals also shows a positive effect on the stable component. Third, the results of the *rRMSE* also show that the ASV-MIDAS model outperforms the traditional SV model. Moreover, the in-sample performance in China's stock market using the ASV-MIDAS model improves much more than that in the U.S. stock

[7] We consider the volatility of a macroeconomic variable. Empirically, the volatility of a macroeconomic variable has better performance than the level of the macroeconomic variable in mixed frequency volatility model specifications (Zheng and Shang, 2014). According to Blanchard and Simon (2001) and Liu and Liu (2005), we use the rolling window method to calculate time-varying volatility.

[8] The SV-MIDAS model is expressed by Eqs.(7)-(11). For example, we just change Eq. (9) $\ln\tau_t = m + \theta\sum_{p=1}^{P}\phi_p(\omega_1, \omega_2)X_{t-p}^n$ to $\ln\tau_t = m + \theta\sum_{p=1}^{24}\phi_p(\omega_1,\omega_2)CPI_{t-p}^n$, then have the SVMIDAS(2)-CPI model.

[9] Prior distribution of $\rho$is that $\rho\tilde{}U(-1,1)$.For the specification of the posterior distribution, please refer to Nakajima and Omori (2009).

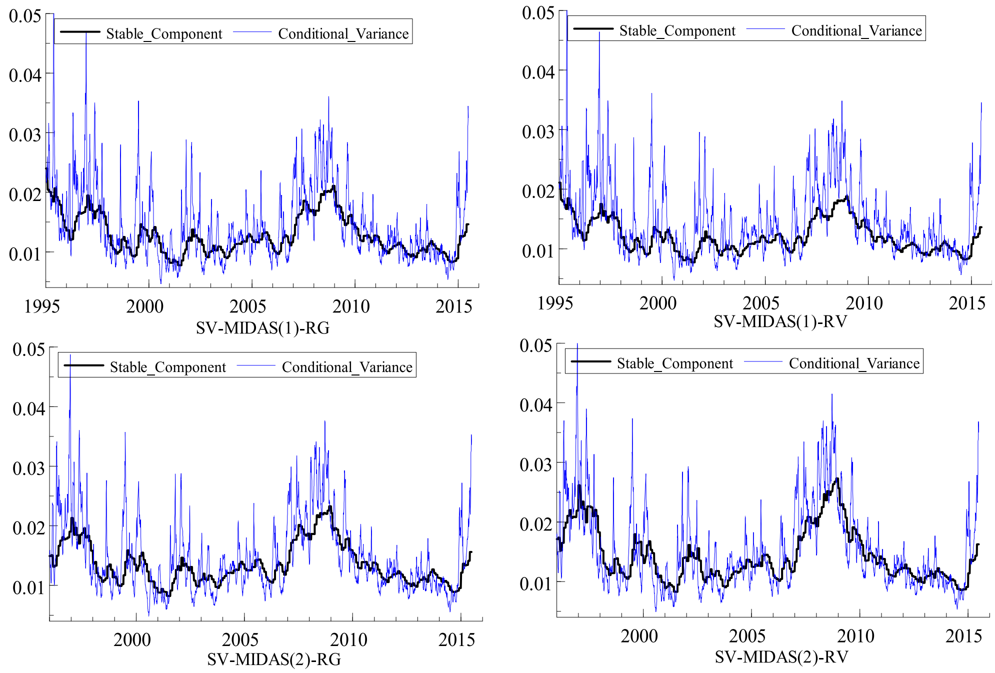

Fig. 2. Estimated daily conditional variance and its stable component (Shanghai Composite Index).

Table 4
Estimated results of the SV-MIDAS model (S&P 500).

	BSV	SV-MIDAS (1)-RG	SV-MIDAS (2)-RG	SV-MIDAS (1)-RV	SV-MIDAS (2)-RV
φ	0.9870	–	–	–	–
	[0.9810, 0.9923]				
$\tilde{\varphi}$	–	0.9606	0.9493	0.9582	0.9569
		[0.9471, 0.9724]	[0.9323, 0.9639]	[0.9487, 0.9712]	[0.9425, 0.9696]
μ	−9.4148	–	–	–	–
	[-9.7505,-9.0869]				
σ_η	0.1483	0.1928	0.2108	0.1945	0.1987
	[0.1260, 0.1735]	[0.1642, 0.2244]	[0.1787, 0.2465]	[0.1654, 0.2307]	[0.1695, 0.2321]
m	–	−10.1565	−9.9077	−10.0607	−10.1296
		[-10.1957, −10.0721]	[-10.0164, −9.8550]	[-10.1201, −10.0205]	[-10.2246, −10.0234]
θ	–	0.9945	0.7422	0.8833	0.8574
		[0.9560, 1.0456]	[0.5820, 0.8626]	[0.8436, 0.9311]	[0.7969, 0.9125]
ω_1	–	2.4099	8.5833	2.2735	5.6079
		[2.3487, 2.4908]	[8.5179, 8.6473]	[2.2017, 2.3802]	[5.5664, 5.6484]
rRMSE	–	0.9226	0.9092	0.9501	0.9081

Note:BSV is the traditional SV model. The reported parameter estimates are means of the posterior distribution. The value in parentheses is the 95% confidence interval. *rRMSE* is the relative *RMSE* for a comparison of the in-sample fitting between a SV-MIDAS model and the traditional SV model.

market.

4.4. Comparison of out-of-sample forecasts

Now let us discuss the out-of-sample forecast performance of the SV-MIDAS and ASV-MIDAS models with different low-frequency variables. Referring to Shang and Zheng (2018), we use the rolling window approach to compute the results of the out-of-sample forecasts. Specifically, using the data samples of a fixed window, we estimate the model parameters and compute the one-step-ahead forecast values of daily volatility. We set a forecast horizon from January 2, 2012 to June 30, 2015, for which the sizes of the forecast samples for China and the United States are 845 and 877, respectively. We use the *rRMSE* to evaluate the performance of the out-of-sample forecasts of the SV-MIDAS and ASV-MIDAS models.

Table 7 reports the rolling window forecast results for all types of mixed-frequency SV models. On the one hand, we find that all SV-MIDAS models can improve forecast precision in the Chinese stock market, except for the SV-MIDAS (2)-M1 model. Among the SV-MIDAS models with a macroeconomic variable, the SV-MIDAS (2)-Lead model has the best performance. This result suggests that the leading index is an important source of volatility in the Chinese stock market. In contrast to in-sample fitting results, the out-of-sample forecast results for the ASV-MIDAS model are inferior to those of the SV-MIDAS model. On the other hand, for the U.S. stock market, the forecast results of the SV-MIDAS model are similar to those of the Chinese stock market. The SV-MIDAS (2)-Lead model also has the best performance among the SV-MIDAS models with macroeconomic variables. Similarly, the out-of-sample forecast results for the ASV-MIDAS model are worse than those of the SV-MIDAS model. Comparing the values of the *rRMSE* of both SV-MIDAS and ASV-MIDAS models, we find that the *rRMSE* in the Chinese stock market is less than that of the U.S. stock market. Overall, the forecast results of all types of mixed-frequency SV models are better for

Table 5
Estimated results of SV-MIDAS model with macro variables.

	Shanghai Composition Index			SP500 Index		
	SV-MIDAS (2)-Lead	SV-MIDAS (2)-M1	SV-MIDAS (2)-CPI	SV-MIDAS (2)-Lead	SV-MIDAS (2)-M1	SV-MIDAS (2)-CPI
$\tilde{\varphi}$	0.9943	0.9950	0.9893	0.9969	0.9937	0.9897
	[0.9906, 0.9975]	[0.9918, 0.9978]	[0.9835, 0.9945]	[0.9945, 0.9989]	[0.9901, 0.9969]	[0.9844, 0.9942]
σ_η	0.1751	0.1412	0.1600	0.1512	0.1443	0.1528
	[0.1475, 0.2053]	[0.1178, 0.1669]	[0.1338, 0.1881]	[0.1288, 0.1762]	[0.1239, 0.1666]	[0.1303, 0.1789]
m	−8.8043	−8.7472	−8.7979	−9.9703	−10.0309	−8.6501
	[-8.8914, −8.6756]	[-8.8208, −8.6807]	[-8.859, −8.7376]	[-10.088, −9.8713]	[-10.075, −9.9750]	[-8.7062, −8.5878]
θ	1.0477	0.6631	0.7128	0.7757	0.7812	0.6792
	[0.9716, 1.1112]	[0.6118, 0.7304]	[0.6183, 0.8367]	[0.6988, 0.8260]	[0.6690, 0.8805]	[0.5709, 0.7641]
ω_1	6.6175	3.9808	3.9124	2.4409	2.4179	2.4776
	[6.5488, 6.6848]	[3.9406, 4.0180]	[3.8702, 3.9801]	[2.3987, 2.4780]	[2.3312, 2.4967]	[2.4338, 2.5178]
rRMSE	0.7974	0.8974	0.9503	0.9688	1.0254	0.9781

Note: The reported parameter estimates are the means of the posterior distribution. The value in parentheses represents the 95% confidence interval. SV-MIDAS(2)-Lead, SV-MIDAS(2)-M1, and SV-MIDAS(2)-CPI represent the SV-MIDAS model with the Composite Leading Indicator, M1, and CPI, respectively. rRMSE is the relative RMSE for a comparison of the in-sample fit performances between a SV-MIDAS model and the traditional SV model.

Table 6
Estimated results of ASV-MIDAS model.

Panel A: Shanghai Composite Index					
	ASV-MIDAS (2)-RG	ASV-MIDAS (2)-RV	ASV-MIDAS (2)-Lead	ASV-MIDAS (2)-M1	ASV_MIDAS (2)_CPI
σ_η	0.2983	0.2986	0.2092	0.2051	0.2407
	[0.2549, 0.3480]	[0.2558, 0.3471]	[0.1802, 0.2423]	[0.1767, 0.2366]	[0.2028, 0.2838]
$\tilde{\varphi}$	0.9403	0.9419	0.9912	0.9928	0.9759
	[0.9218, 0.9585]	[0.9237, 0.9576]	[0.9861, 0.9956]	[0.9890, 0.9963]	[0.9650, 0.9851]
m	−8.7424	−8.9975	−9.2091	−9.0470	−9.0900
	[-8.7663, −8.6189]	[-9.0324, −8.8577]	[-9.2375, −9.0725]	[-9.1053, −8.8666]	[-9.1063, −9.0048]
θ	0.8539	0.8603	0.8234	0.7862	0.6989
	[0.7728, 0.8780]	[0.8456, 0.8744]	[0.8012, 0.9155]	[0.6898, 0.8213]	[0.6877, 0.7098]
ω_1	7.1970	6.7952	6.3774	3.6931	6.3440
	[7.1617, 7.2161]	[6.7624, 6.8154]	[6.3510, 6.4152]	[3.6647, 3.7324]	[6.2977, 6.3767]
ρ	−0.2220 [-0.3007,	−0.2183	−0.1691	−0.1916	−0.1666
	−0.1413]	[-0.2967, −0.1373]	[-0.2622, −0.0772]	[-0.2797, −0.1007]	[-0.2527, −0.0770]
rRMSE	0.6930	0.6925	0.7214	0.7217	0.7089
Panel B: S&P 500 Index					
	ASV-MIDAS (2)-RG	ASV-MIDAS (2)-RV	ASV-MIDAS (2)-Lead	ASV-MIDAS (2)-M1	ASV_MIDAS (2)_CPI
σ_η	0.2318	0.2219	0.1918	0.1711	0.1814
	[0.2033, 0.2628]	[0.1945, 0.2509]	[0.1674, 0.2169]	[0.1499, 0.1926]	[0.1578, 0.2057]
$\tilde{\varphi}$	0.9607	0.9697	0.9876	0.9953	0.9897
	[0.9511, 0.9688]	[0.9619, 0.9763]	[0.9828, 0.9917]	[0.9935, 0.9969]	[0.9857, 0.9932]
m	−9.8819	−9.9602	−9.0930	−8.9301	−9.2629
	[-9.9249, −9.7372]	[-10.024, −9.8055]	[-9.1168, −9.0204]	[-8.9564, −8.8673]	[-9.2843, −9.2339]
θ	0.7949	0.8004	0.7395	0.7557	0.8325
	[0.6952, 0.8273]	[0.7114, 0.8438]	[0.7248, 0.7547]	[0.7173, 0.7738]	[0.8180, 0.8468]
ω_1	7.4097	6.3990	4.3798	2.3420	2.5526
	[7.3740, 7.4461]	[6.3815, 6.4230]	[4.3456, 4.4107]	[2.3180, 2.3622]	[2.5236, 2.5805]
ρ	−0.7552	−0.7259	−0.6705	−0.7065	−0.7028
	[-0.8064, −0.6937]	[-0.7789, −0.6626]	[-0.7349, −0.5952]	[-0.7652, −0.6375]	[-0.7636, −0.6327]
rRMSE	0.7777	0.7758	0.7784	0.7879	0.7850

Note: The reported parameter estimates are means of the posterior distribution. The value in parentheses represents the 95% confidence interval. Panel A uses samples of the Shanghai Composite Index. Panel B uses samples of the S&P 500 index.

the Chinese market than for the U.S. market.

Furthermore, we test the forecast performance of SV-MIDAS model and ASV-MIDAS model via Diebold and Mariano (1995)'s statistics[10]. The null hypothesis of Diebold-Mariano (D-M) test is that there is the same mean squared error for the two forecasts. This paper will compute the D-M statistics and the corresponding results are shown in Table 8.

As reported in Table 8, the positive values of D-M statistics indicate the better out of sample forecast performance of our mixed-frequency SV model. Comparing with Table 8, we find that the SV-MIDAS models can significantly improve forecast precision in the Chinese stock market, except for the SV-MIDAS (2)-M1 model. Similarly, the SV-MIDAS models also have better out of simple performance in the US stock market. It means that the SV-MIDAS models are significant superiority of the traditional SV model's forecasts. However, Diebold-Mariano statistics shows that the ASV-MIDAS models can't significantly better than the corresponding ASV model's forecasts.

5. Conclusion

In this paper, we develop a mixed-frequency SV model with macroeconomic variables and further extend it to the ASV-MIDAS model by including a leverage effect. In terms of volatility decomposition, the volatility components in the SV-MIDAS and ASV-MIDAS models are decomposed into two components: a stable component and a stochastic

[10] Diebold-Mariano test is used to determine whether forecasts are significantly different. The Diebold-Mariano statistic is constructed by the residuals for the two different forecasts. Under the null hypothesis that the two forecasts are not different, D-M statistic follows a standard normal distribution: D-M ~ N(0, 1).

Table 7
Results of out-of-sample forecasting.

Shanghai Composite Index				S&P 500 Index			
SV-MIDAS	*rRMSE*	ASV-MIDAS	*rRMSE*	SV-MIDAS	*rRMSE*	ASV-MIDAS	*rRMSE*
SV-MIDAS (2)-RG	**0.5102**	ASV-MIDAS (2)-RG	**0.5685**	SV-MIDAS (2)-RG	**0.5782**	ASV-MIDAS (2)-RG	**0.9420**
SV-MIDAS (2)-RV	**0.5601**	ASV-MIDAS (2)-RV	**0.9546**	SV-MIDAS (2)-RV	**0.7335**	ASV-MIDAS (2)-RV	1.0489
SV-MIDAS (2)-Lead	**0.6827**	ASV-MIDAS (2)-Lead	**0.9800**	SV-MIDAS (2)-Lead	**0.7768**	ASV-MIDAS (2)-Lead	1.0838
SV-MIDAS (2)-M1	1.1441	ASV-MIDAS (2)-M1	1.0574	SV-MIDAS (2)-M1	1.7765	ASV-MIDAS (2)-M1	1.4748
SV-MIDAS (2)-CPI	**0.7061**	ASV-MIDAS (2)-CPI	**0.9648**	SV-MIDAS (2)-CPI	**0.8300**	ASV-MIDAS (2)-CPI	1.1332

Note:The boldface indicates that the corresponding model has better performance in out-of-sample forecasting.

Table 8
D-M test of out-of-sample forecasting.

Shanghai Composite Index				S&P 500 Index			
SV-MIDAS	D-M	ASV-MIDAS	D-M	SV-MIDAS	D-M	ASV-MIDAS	D-M
SV-MIDAS (2)-RG	**5.1541*****	ASV-MIDAS (2)-RG	**2.4649****	SV-MIDAS (2)-RG	**19.9913*****	ASV-MIDAS (2)-RG	1.3557
SV-MIDAS (2)-RV	**5.1256*****	ASV-MIDAS (2)-RV	0.7900	SV-MIDAS (2)-RV	**13.9283*****	ASV-MIDAS (2)-RV	−0.2783
SV-MIDAS (2)-Lead	**3.8208*****	ASV-MIDAS (2)-Lead	0.6232	SV-MIDAS (2)-Lead	**4.2799*****	ASV-MIDAS (2)-Lead	−2.8110
SV-MIDAS (2)-M1	−3.4722	ASV-MIDAS (2)-M1	−2.9019	SV-MIDAS (2)-M1	−31.0043	ASV-MIDAS (2)-M1	−9.8688
SV-MIDAS (2)-CPI	**3.3055*****	ASV-MIDAS (2)-CPI	0.6949	SV-MIDAS (2)-CPI	**5.1515*****	ASV-MIDAS (2)-CPI	−3.8954

Note:D-M is Diebold-Mariano statistics. The boldface indicates that the corresponding model is significant better than the basic model in out-of-sample forecasting. * denote significance at the 10% level. ** denote significance at the 5% level. *** denote significance at the 1% level.

component. The stable component is crucial to link the low-frequency variable, such as macroeconomic variable, to stock volatility. We employ the MCMC method to realize the parameter estimation of both the SV-MIDAS model and the ASV-MIDAS model by approximating it to a mixture approximation model.

With an empirical investigation on the Chinese and U.S. stock markets, we can draw the following conclusions. First, the SV-MIDAS model and its extension can describe the time-varying stable component, which is useful to study the volatility mechanism and improve volatility forecasting. We find that the volatility of the macroeconomic fundamentals has a positive effect on this time-varying stable component. It suggests that the SV-MIDAS model can identify the macroeconomic volatility source of stock volatility.

Second, the SV-MIDAS model outperforms the traditional SV model in the in-sample and out-of-sample performances. In particular, among the macroeconomic variables, the Composite Leading Indicator has the best performance in terms of fitting and forecasting the stock volatilities. This suggests that the macroeconomic fundamental helps to fit and forecast the stock market volatility. Moreover, the SV-MIDAS model improves much more for China than for the United States in the in-sample and out-of-sample performances.

Third, the results of the ASV-MIDAS model show that both the Chinese and the U.S. stock markets have significant leverage effects. However, the leverage effect is weaker in China. The ASV-MIDAS model shows the better in-sample fitting results than the corresponding benchmark model. In the presence of leverage effect, the volatility of the macroeconomic fundamentals also contributes to the stable component. Nevertheless, the out-of-sample forecast performance for the ASV-MIDAS model is inferior to the SV-MIDAS model.

In summary, the SV-MIDAS and extended ASV-MIDAS models with macro variables are promising methods for improving the in-sample fitting and out-of-sample predictions. The forecast results of mixed frequency SV model provide practical implications for portfolio investment and risk management. These mixed frequency SV models can also be applied to forecast the volatility of other financial assets such futures and options.

Appendix. MCMCalgorithm

1. Sample$\beta|s,h,y^*$

To sample $\beta=\{\varphi,\sigma_\eta\}$ from the posterior distribution $\pi(\beta|s,h,y^*)\propto f(y^*|\beta,s,h)\pi(\beta)$with the M-H algorithm. First, use a Kalman filter to compute $f(y^*|\beta,s,h)$; second, we obtain $\widehat{\beta}=\{\widehat{\varphi},\widehat{\sigma}_\eta\}$, which maximizes the posterior probability density $\pi(\beta|s,h,y^*)$. We generate a candidate $\beta^*$from a normal distribution $N(\beta_*,\Sigma_*)$truncated over the region $R=\{\beta:|\varphi|<1,\sigma>0\}$, where

$$\beta_*=\widehat{\beta}+\Sigma_*\frac{\partial\log\pi(\beta|s,h,y^*)}{\partial\beta}\bigg|_{\beta=\widehat{\beta}},\ \Sigma_*^{-1}=-\frac{\partial^2\log\pi(\beta|s,h,y^*)}{\partial\beta\partial\beta'}\bigg|_{\beta=\widehat{\beta}} \tag{26}$$

Let β take the current valueβ_0; we accept the candidate $\beta^*$with the probability

$$\alpha(\beta_0,\beta^*|s,h,y^*)=min\left\{\frac{\pi(\beta^*|s,h,y^*)f_N(\beta_0|(\beta_*,\Sigma_*))}{\pi(\beta_0|s,h,y^*)f_N(\beta^*|(\beta_*,\Sigma_*))},1\right\} \tag{27}$$

2. Sample $h|s,\beta,y^*$

Referring to Kim et al. (1998), we compute an augmented Kalman filter based on an approximating Gaussian state space model. Then, we sample

$h|s,\beta,y^*$ using the simulation smoother (Durbin and Koopman, 2002). Given $s_t = k$, the approximating linear Gaussian state space model can be expressed as

$$\begin{pmatrix} y^*_{i,t} \\ h_{i,t+1} \end{pmatrix} = \begin{pmatrix} c_{i,t} \\ d_{i,t} \end{pmatrix} + \begin{pmatrix} Z_{i,t}h_{i,t} \\ T_{i,t}h_{i,t} \end{pmatrix} + \begin{pmatrix} G_{i,t}u_{i,t} \\ H_{i,t}u_{i,t} \end{pmatrix} \quad (28)$$

$$h_1 | y^*_0 \sim N\left(a_{1|0}, P_{1|0}\right) \quad (29)$$

where$c_{i,t} = m_k Z_{i,t} = 1 G_{i,t} = (\nu_k, 0) d_{i,t} = 0 T_{i,t} = \varphi$, $H_{i,t} = (0, \sigma_\eta) u_{i,t} \sim N(0, I_2) a_{1|0} = 0, P_{1|0} = \sigma^2_\eta/(1-\varphi^2)$

The Kalman filter can be written as:

$$\begin{aligned} h_{i+1,t|i,t} &= d_{i,t} + T_{i,t}h_{i,t|i-1,t} + K_{i,t}\upsilon_{i,t} \\ P_{i+1,t|i,t} &= T_{i,t}P_{i,t|i-1,t}L'_{i,t} + G_{i,t}G'_{i,t} \\ \upsilon_{i,t} &= y^*_{i,t} - Z_{i,t}h_{i,t|i-1,t} - c_{i,t} \\ F_{i,t} &= Z_{i,t}P_{i,t|i-1,t}Z'_{i,t} + H_{it}(H_{it} - K_{i,t}G_{i,t})' \\ K_{i,t} &= T_{i,t}P_{i,t|i-1,t}Z'_{i,t}F^{-1}_{i,t} \\ L_{i,t} &= T_{i,t} - K_{i,t}Z_{i,t} \end{aligned} \quad (30)$$

Referring to Durbin and Koopman (2002), using the simulation smoother to sample$h|s,\beta,y^*$, set$r_n = 0, N_n = 0.D_{i,t} = F^{-1}_{i,t} + K'_{i,t}N_{i,t}K_{i,t}, n_{i,t} = F^{-1}_{i,t}\upsilon_{i,t} - K'_{i,t}r_{i,t}$. For $\{i,t\} = n,\cdots,1$, we have the following recursion:

$$\begin{aligned} C_{i,t} &= G_{i,t}G'_{i,t} - G_{i,t}G'_{i,t}D_{i,t}G_{i,t}G'_{i,t} \\ \kappa_{i,t} &\sim N(0, C_{i,t}) \\ V_{i,t} &= G_{i,t}G'_{i,t}\left(D_{i,t}Z_{i,t} - K'_{i,t}N_{i,t}T_{i,t}\right) \\ r_{i-1,t} &= Z'_{i,t}F^{-1}_{i,t}\upsilon_{i,t} + L_{i,t}r_{i,t} - V'_{i,t}C^{-1}_{i,t}\kappa_{i,t} \\ N_{i-1,t} &= Z'^{\prime}_{i,t}F^{-1}_{i,t}Z'_{i,t} + L'_{i,t}N_{i,t}L_{i,t} - V'_{i,t}C^{-1}_{i,t}V_{i,t} \end{aligned} \quad (31)$$

We take the value of$y^*_{i,t}$-$G_{i,t}G'_{i,t}n_{i,t}$-$\kappa_{i,t}$as the sample result of $c_{i,t} + Z_{i,t}h_{i,t}$.

3. Sample $s|\beta, h, y^*$

To sample s_t, for any $k = 1,\cdots,K$, we need to compute the following posterior distribution:

$$\pi(s_t = k|\beta, h, y^*) \propto q_k \frac{1}{\nu_{i,t}} exp\left\{\frac{\left(y^*_{i,t} - m_k - h_{i,t}\right)^2}{2\nu^2_{i,t}}\right\} exp\left\{-\frac{(h_{i+1,t} - \varphi h_{i,t})^2}{2\sigma_\eta{}^2}\right\} \quad (32)$$

for any date sample$\{i,t\} = 1,\cdots,n$, we need to sample $s_t$from K independent discrete distributions.

4. Sample $\{m,\theta,\omega\}|\beta, s, h, y$

Referring to Chib and Greenberg (1995) and Koop (2003), we sample $\gamma = \{m,\theta,\omega\}$ using a random-walk M-H algorithm. We draw the candidate$\gamma^*$by the following equation:

$$\gamma^* = \gamma + c^2 * z \quad (33)$$

where c is a scale parameter that takes value of 0.1, $z$is an increment that is sampled from Student's t-distribution. The posterior distribution of γcan be written as

$$\pi(\gamma|\beta, h, s, y^*) \propto \sum_{k=1}^{K} q_k \frac{1}{\nu_{i,t}} exp\left\{\frac{\left(y_{i,t} - \tau_t(\gamma) - m_k - h_{i,t}\right)^2}{2\nu^2_{i,t}}\right\} exp\left\{-\frac{(h_{i+1,t} - \varphi h_{i,t})^2}{2\sigma^2_\eta}\right\} \quad (34)$$

where $\tau_t(\gamma) = m + \theta \sum_{k=1}^{K} \phi_k(\omega_1, \omega_2) RV_{t-k}$.Let γtake the current value γ_0; we accept the candidate $\gamma^*$with the probability

$$\alpha(\gamma_0, \gamma^*|\beta, s, h, y^*) = min\left\{\frac{\pi(\gamma^*|\beta, s, h, y^*)}{\pi(\gamma_0|\beta, s, h, y^*)}, 1\right\} \quad (35)$$

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.econmod.2020.03.013.

References

Alizadeh, S., Brandt, M.W., Diebold, F.X., 2002. Range-based estimation of stochastic volatility models. J. Finance 57, 1047–1091.
Asgharian, H., Hou, A.J., Javed, F., 2013. The importance of the macroeconomic variables in forecasting stock return variance: a GARCH-MIDAS Approach. J. Forecast. 32 (7), 600–612.
Bansal, R., Yaron, A., 2004. Risks for the long-run: a potential resolution of asset pricing puzzles. J. Finance 59, 1481–1509.
Becker, R., Clements, A., 2007. Forecasting stock market volatility conditional on macroeconomic conditions. National Centre for Econometric Research (NCER) Working Paper Series, (18) 1–33.
Blanchard, O., Simon, J., 2001. The long and large decline in US output volatility. Brookings Pap. Econ. Activ. 135–174, 2001 2001(1).
Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. J. Econom. 31 (3), 307–327.
Chan, J.C.C., Grant, A.L., 2016. Modeling energy price dynamics: GARCH versus stochastic volatility. Energy Econ. 54, 182–189.
Chen, J., Xiong, X., Zhu, J., Zhu, X., 2017. Asset prices and economic fluctuations: the implications of stochastic volatility. Econ. Modell. 64, 128–140.
Chib, S., Greenberg, E., 1995. Understanding the metropolis-hastings algorithm. Am. Statistician 49 (4), 327–335.
Diebold, F X, Mariano, R S, 1995. Comparing predictive accuracy. J. Bus. Econ. Stat. 13 (3), 253–263.
Ding, L., Vo, M., 2012. Exchange rates and oil prices: a multivariate stochastic volatility analysis. Q. Rev. Econ. Finance 52 (1), 15–37.
Durbin, J., Koopman, S.J., 2002. A simple and efficient simulation smoother for state space time series analysis. Biometrika 89 (3), 603–616.
Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50, 987–1007.
Engle, R., Lee, G., 1999. A permanent and transitory component model of stock return volatility. In: Engle, R., White, H. (Eds.), Cointegration, Causality, and Forecasting: A Festschrift in Honor of Clive W. J. Granger. Oxford University Press, pp. 475–497.
Engle, R.F., Ghysels, E., Sohn, B., 2013. Stock market volatility and macroeconomic fundamentals. Rev. Econ. Stat. 95 (3), 776–797.
Engle, R.F., Rangel, J.G., 2008. The spline-GARCH model for low-frequency volatility and its global macroeconomic causes. Rev. Financ. Stud. 21 (3), 1187–1222.
Ghysels, E., Santa-Clara, P., Valkanov, R., 2004. The MIDAS Touch: Mixed Data Sampling Regressions." Mimeo. Chapel Hill, N. C.
Hamilton, J.D., Lin, G., 1996. Stock market volatility and the business cycle. J. Appl. Econom. 11 (5), 573–593.
Harvey, A., Ruiz, E., Shephard, N., 1994. Multivariate stochastic variance models. Rev. Econ. Stud. 61 (2), 247–264.
Harvey, A.C., Shephard, N., 1996. Estimation of an asymmetric stochastic volatility model for asset returns. J. Bus. Econ. Stat. 14 (4), 429–434.
Jensen, M.J., Maheu, J.M., 2010. Bayesian semiparametric stochastic volatility modeling. J. Econom. 157 (2), 306–316.
Jiang, X.F., Zheng, B., Ren, F., et al., 2017. Localized motion in random matrix decomposition of complex financial systems. Phys. Stat. Mech. Appl. 471, 154–161.
Kanaya, S., Kristensen, D., 2016. Estimation of stochastic volatility models by nonparametric filtering. Econom. Theor. 32 (4), 861–916.
Kastner, G., Frühwirth-Schnatter, S., Lopes, H.F., 2017. Efficient Bayesian inference for multivariate factor stochastic volatility models. J. Comput. Graph Stat. 26 (4), 905–917.
Kim, S., Shephard, N., Chib, S., 1998. Stochastic volatility: likelihood inference and comparison with ARCH models. Rev. Econ. Stud. 65 (3), 361–393.
Koop, G.M., 2003. Bayesian Econometrics. John Wiley & Sons Inc.
Liu, J.Q., Liu, Z.G., 2005. The analysis of dynamic patterns and resources of output volatilities in China's business cycles. Econ. Res. J. 3, 26–35 ([in Chinese]).
Nakajima, J., Omori, Y., 2009. Leverage, heavy-tails and correlated jumps in stochastic volatility models. Comput. Stat. Data Anal. 53 (6), 2335–2353.
Nelson, D.B., 1991. Conditional heteroskedasticity in asset returns: a new approach. Econometrica 347–370.
Omori, Y., Chib, S., Shephard, N., et al., 2007. Stochastic volatility with leverage: fast and efficient likelihood inference. J. Econom. 140 (2), 425–449.
Ouyang, F.Y., Zheng, B., Jiang, X.F., 2014. Spatial and temporal structures of four financial markets in Greater China. Phys. Stat. Mech. Appl. 402, 236–244.
Parkinson, M., 1980. The extreme value method for estimating the variance of the rate of return. J. Bus. 61–65.
Peiris, S., Asai, M., McAleer, M., 2017. Estimating and forecasting generalized fractional long memory stochastic volatility models. J. Risk Financ. Manag. 10 (4), 23.
Sandmann, G., Koopman, S.J., 1998. Estimation of stochastic volatility models via Monte Carlo maximum likelihood. J. Econom. 87 (2), 271–301.
Shang, Y., Liu, L., 2017. An extension of stochastic volatility model with mixed frequency information. Econ. Lett. 155, 144–148.
Shang, Y., Zheng, T., 2018. Fitting and forecasting yield curves with a mixed-frequency affine model: evidence from China. Econ. Modell. 68, 145–154.
Shen, J., Zheng, B., 2009. On return-volatility correlation in financial dynamics. EPL (Europhysics Letters) 88 (2), 28003.
Shyu, Y.W., Hsia, K., 2008. Does stock market volatility with regime shifts signal the business cycle in Taiwan? Int. J. Electron. Finance 2 (4), 433–450.
Takahashi, M., Watanabe, T., Omori, Y., 2016. Volatility and quantile forecasts by realized stochastic volatility models with generalized hyperbolic distribution. Int. J. Forecast. 32 (2), 437–457.
Taylor, S.J., 1994. Modeling stochastic volatility: a review and comparative study. Math. Finance 4 (2), 183–204.
Wachter, J., 2013. Can time-varying risk of rare disasters explain aggregate stock market volatility? J. Finance 68 (3), 987–1035.
Yu, J., 2005. On leverage in a stochastic volatility model. J. Econom. 127 (2), 165–178.
Zheng, T.G., Wang, X., 2013. Measuring China's business cycle with mixed-frequency data and its real time analysis. Econ. Res. J. 6, 58–70 ([in Chinese]).
Zheng, T.G., Shang, Y.H., 2014. Measuring and forecasting the stock market volatility based on macroeconomic fundamentals. The Journal of World Economy 12, 118–139 ([in Chinese]).